Mathematics

Foundation

Unit 1

Number

Place Value and Reading and Writing Numbers

Place value is the value given to a digit by its place in a number.

Ascending means smallest to biggest; descending means biggest to smallest

Place Value Table

Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
		2	3	4	0	7

Example: What is the value of the 8 in the number 904, 860?

Using the place value table:

Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
	9	0	4	8	6	0

The 8 is in the hundreds column, so the value of the 8 is: 8 hundred or 800

Ordering Numbers

Example: Put the following numbers in ascending order 4385, 4380, 4290

Use the place value table to compare the numbers:

Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
			4	3	8	5
			4	3	8	0
			4	2	9	0

Ascending means smallest to biggest, so we need the smallest number first. Start by looking at the numbers in the far-left column, in this case the thousands column. These numbers are all 4's so look in the next column, the hundreds column. 2 is the smallest number here so 4290 comes first. Then look in the tens column, both the numbers left are 8's so we look in the ones column, 0 is the smallest number here so 4380 comes second.

The order is: 4290, 4380, 4385

Reading and Writing Numbers

We can use the place value table to help read and write large numbers in words and in figures.

Example: Write the number sixty thousand and nine in figures.

Using the place value table:

Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
		6	0	0	0	9

Sixty thousand and nine

The number sixty thousand and nine in figures is: 60, 009

Example: Write the number 105 332 in words.

Using the place value table:

Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
	1	0	5	3	3	2

One hundred and five thousand three hundred and thirty-two

The number 105 332 in words is: One hundred and five thousand, three hundred and thirty-two

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Unit 1

Subtracting Whole Numbers

When subtracting whole numbers, we need to line up the digits at the right-hand side, ones in the ones column, tens in the tens column, etc. If the number we are subtracting from is smaller than the number we have, then we will need to "borrow" from the next number.

Example: 364 – 128

$$\begin{array}{r} 3\,\overset{5}{\cancel{6}}\,{}^{1}4 \\ -\;1\,2\,8 \\ \hline 6 \end{array} \qquad \begin{array}{r} 3\,\overset{5}{\cancel{6}}\,{}^{1}4 \\ -\;1\,2\,8 \\ \hline 3\,6 \end{array} \qquad \begin{array}{r} 3\,\overset{5}{\cancel{6}}\,{}^{1}4 \\ -\;1\,2\,8 \\ \hline 2\,3\,6 \end{array} \qquad \text{So } 364 - 128 = \underline{\mathbf{236}}.$$

Adding Whole Numbers

When adding whole numbers, we need to line up the digits at the right-hand side, ones in the ones column, tens in the tens column, etc.

Example: 145 + 28

$$\begin{array}{r} 1\,4\,5 \\ +\;\;\,2\,8 \\ \hline {}_{1}3 \end{array} \qquad \begin{array}{r} 1\,4\,5 \\ +\;\;\,2\,8 \\ \hline 7\,{}_{1}3 \end{array} \qquad \begin{array}{r} 1\,4\,5 \\ +\;\;\,2\,8 \\ \hline 1\,7\,{}_{1}3 \end{array} \qquad \text{So } 145 + 28 = \underline{\mathbf{173}}.$$

Dividing Whole Numbers:

Example – short division: 3144 ÷ 8

Using short division:

1. 8 doesn't go into 3, so look at the first two digits.
2. 8 goes into 31 three times, with remainder 7.
3. 8 goes into 74 nine times, with remainder 2.
4. 8 goes into 24 three times exactly.

$$\begin{array}{r} 0\;3\;\;\;\;\;\; \\ 8\overline{)3\;1\;{}^{7}4\;4} \end{array} \qquad \begin{array}{r} 0\;3\;9\;\;\; \\ 8\overline{)3\;1\;{}^{7}4\;{}^{2}4} \end{array} \qquad \begin{array}{r} 0\;3\;9\;3 \\ 8\overline{)3\;1\;{}^{7}4\;{}^{2}4} \end{array}$$

So 3144 ÷ 8 = **393**.

Example – long division: 782 ÷ 34

```
        23      (answer line)
34   | 782
    -  680     (34 x 20 = 680, put 2 in the tens column on the answer line)
       102
    -  102     (34 x 3 = 102, put 3 in the units column on the answer line)
       000
```

Therefore 782 ÷ 34 = 23

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Unit 1

Multiplying Whole Numbers

Note: 12 x 3 is the same as 3 x 12

Example 1 – short multiplication: 127 x 6

```
  1 2 7 ×
      6
  -----
  7 6 2
  -----
  1 4
```

1st work out 7 × 6 = 42 Write down the 2 units in the box and carry the 4 tens under the tens column

Next work out 2 × 6 = 12 (or 20 × 6) you need to add on the carry of 4 (or 40) to make 16 (or 160). Place 6 (or 6 tens) in the box in the tens column and carry the 1(or 100) in the hundreds column

Lastly, work out 1 × 6 (or 100 × 6) and add on the carry of 1 (or 100) to make 7 (or 700). Place 7 in the box in the hundreds column

Example 2 – long multiplication: 352 x 27

Method 1: Column Method

```
    3 5 2 ×
      2 7
  -------
  2 4 6 4 +
  7 0 4 0
  -------
  9 5 0 4
  1
```

352 x 7 = 2464

Put a 0 in the ones column as we are now multiplying by the number in the tens column 352 x 2 = 704

Add the two rows, 2464 + 7040 = 950

Example 3 – long multiplication: 23 x 34

Method 2: Grid Method

23 is split into 20 and 3

34 is split into 30 and 4

×	20	3
30	600	90
4	80	12

30 x 20 = 600

30 x 3 = 90

4 x 20 = 80

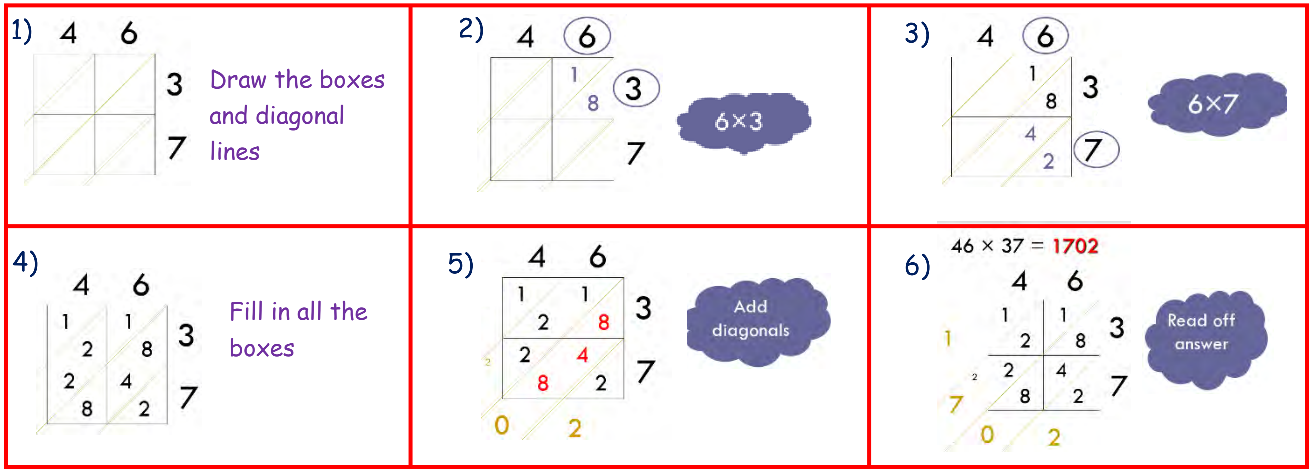
4 x 3 = 12

Add all the answers

600 + 90 + 80 + 12 = 782

Example 4 – long multiplication: 46 x 37

Method 3: Box Method



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Unit 1

BODMAS / BIDMAS

Remember, it must be used like this:

First do any: (Brackets)

Followed by any: Indices

Left to right do any: D÷vision & Mxltiplication

Lastly, left to right: Addi+ion & Subtr-ction

Multiplying and Dividing by Multiples of 10:

When you multiply your digits move left, and when you divide your digits move right. The distance they move depends on the amount of zeros in your number (10, 100, 1000 …). Eg. If you are multiplying by 100 the digits move to the left 2 places because 100 has 2 zeros.

Example 1: 43 x 10 = 430

43 moves one place value to the left (10 has one zero) and the space is filled in with a zero

Example 2: 789 x 1000 = 789000

789 moves three place values to the left (1000 has three zeros) and the spaces are filled in with zeros

Example 3: 3200 ÷ 100 = 32

3200 moves two place values to the right (100 has two zeros),

Example 4: 86 ÷ 10 = 8.6

86 moves one place value to the right (10 has one zero), the 6 moves into the tenths column, the answer is a decimal

BIDMAS / BODMAS:

BIDMAS or BODMAS is a way of helping you to remember the order in which to do your calculations.

Example 1: $2 + \underline{7 \times 10}$

$= 2 + 70$

$= 72$

This question involves addition and multiplication, using the rules of BIDMAS, multiplication is first

Example 2: $\underline{22 - 6} + 4$

$= 16 + 4$

$= 20$

This question involves subtraction and addition, using the rules of BIDMAS, work left to right doing whatever is first

Example 3: $\underline{(4 + 5)}^2 - 4 \times 9$

$= \underline{(9)^2} - 4 \times 9$

$= 81 - \underline{4 \times 9}$

$= 81 - 36$

$= 45$

This question involves multiple operations, just follow the rules of BIDMAS. Brackets first, then indices, then multiplying, then subtraction

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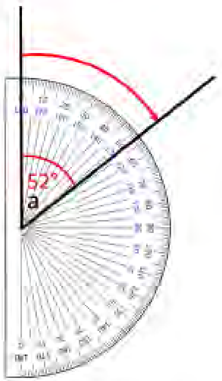
Unit 2

Angles and Basic Angle Properties

Measure angles

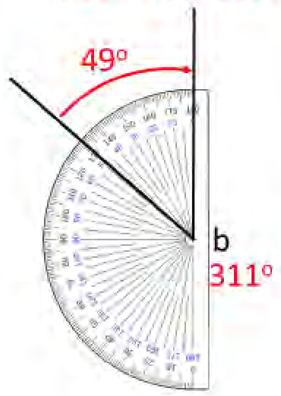


Example: Measure the angle marked a.

1. Place the centre point of the protractor on the corner of the angle and line up the zero line
2. Work out the direction to measure from, make sure you always read from 0°
3. Read the angle from the protractor and label

Measure reflex angles

360 – 49 = 311°

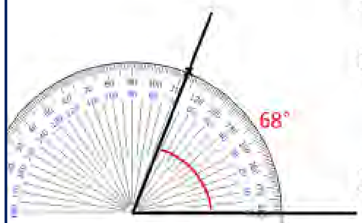


We use the fact that a full turn adds up to 360°.

Example: Measure the angle marked b.

1. Measure the smaller angle that makes up a full turn.
2. Subtract this angle from 360 to calculate the reflex angle then label.

Draw angles

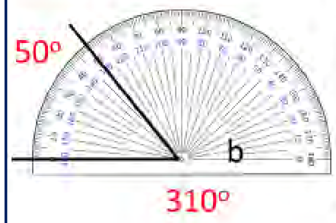


Example: Accurately draw an angle of 68°.

1. Draw a horizontal line
2. Place the centre point of the protractor on one end of the line. Line up the zero of the protractor with the drawn line.
3. Work out the direction, always measure from the zero, and place a mark at 68°.
4. Draw a line from the end of the line you used through the mark and label the angle. **Check using angle types!**

Draw reflex angles

360 – 310 = 50°



We use the fact that a full turn adds up to 360°.

Example: Accurately draw an angle of 310°.

1. Subtract the angle from 360.
2. Draw an angle of this size.
3. Label the opposite angle 310°.

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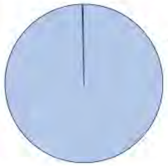
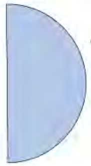
Unit 2

What angle is …



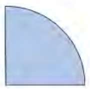
… a full turn?

360°



… a half turn?

180°



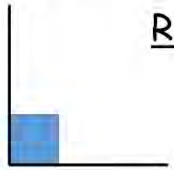
… a quarter turn?

90°

… a three quarter turn?

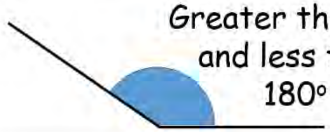
270°

What type of angle is …



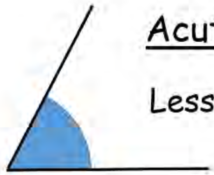
Right-Angle

Exactly 90°



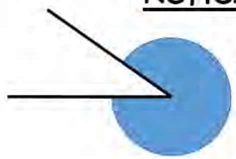
Obtuse Angle

Greater than 90° and less than 180°



Acute Angle

Less than 90°

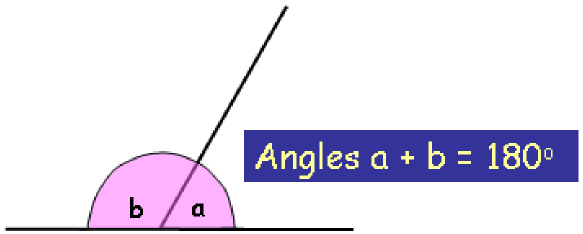


Reflex Angle

Greater than 180° and less than 360°

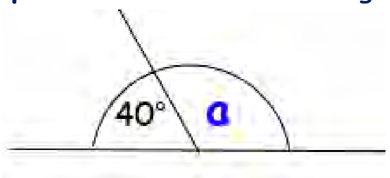
Angles on a Straight Line

Fact: Angles on a straight line add up to 180°



How to spot it: Find any continuous straight line, with another straight line joining it or cutting across it

Example 1: Find the size of angle a

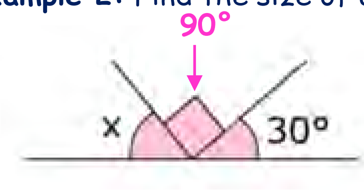


Angles on a straight line add to 180°

$a = 180 - 40$

$a = 140°$

Example 2: Find the size of angle x



Angles on a straight line add to 180°

$90 + 30 = 120$

$x = 180 - 120$

$x = 60°$

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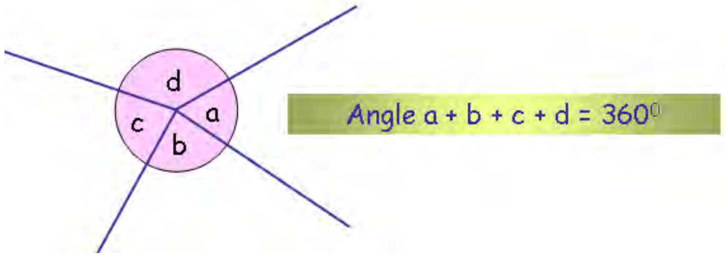
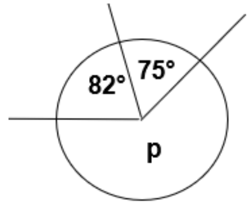
Angles around a Point

Fact: Angles around a point add up to 360°



How to spot it: If you have a collection of lines all crossing at one point, then it is time to use this rule.

Example 1: Find the size of angle p



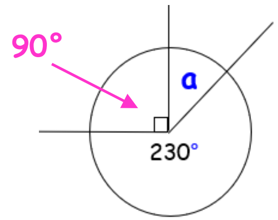
Angles around a point add to 360°

$82 + 75 = 157$

$p = 360 - 157$

$p = 203°$

Example 2: Find the size of angle a



Angles around a point add to 360°

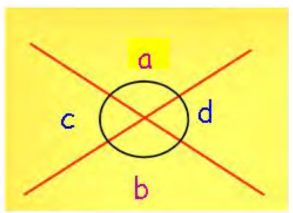
$90 + 230 = 320$

$a = 360 - 320$

$a = 40°$

Opposite Angles

Fact: Opposite Angles are equal



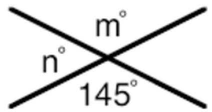
a = b

c = d

How to spot it: Find two continuous straight lines crossing at a point. The pairs of angles opposite each other will be equal

Note: All the angles around that point will add up to 360°

Example 1: Find the size of angles m and n



Opposite angles are equal

$m = 145°$

Angles on a straight line add to 180°

$n = 180 - 145$

$n = 35°$

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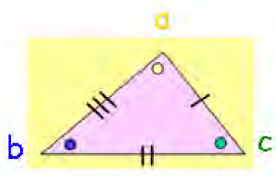
Angles in a Triangle

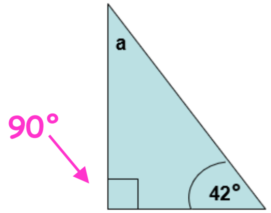
Fact: The interior (inside) angles of a triangle add up to 180°



a + b + c = 180⁰

How to spot it: All the angles inside ANY triangle will add up to 180°

Example 1: Find the size of angle p



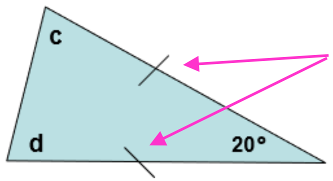
Angles in a triangle add to 180°

$90 + 42 = 132$

$a = 180 - 132$

$a = 48°$

Example 2: Find the size of angles c and d



These two lines mean the sides are equal, it is an isosceles triangle, this means that the two angles, c and d, are also equal.

Angles in a triangle add to 180°

$180 - 20 = 160$

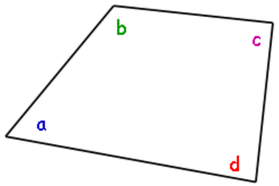
$c = 160 \div 2$

Because the two angles are equal.

$c = 80°$ $\quad d = 80°$

Angles in a Quadrilateral

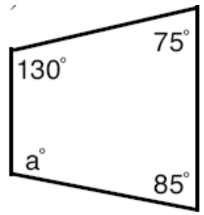
Fact: Interior (inside) angles of a quadrilateral add up to 360°



a + b + c + d = 360⁰

How to spot it: All the angles in any 4-sided shape will add up to 360°

Example 1: Find the size of angle a



Angles in a quadrilateral add to 360°

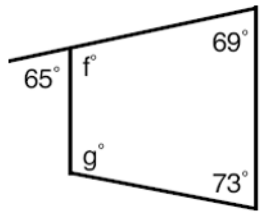
$130 + 75 + 85 = 290$

$a = 360 - 290$

$a = 70°$

Example 2: Find the size of angles f and g

Angle f and 65° are on a straight line



Angles on a straight line add to 180°

$f = 180 - 65$

$f = 115°$

Angles in a quadrilateral add to 360°

$115 + 69 + 73 = 257$

$g = 360 - 257$

$g = 103°$

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Unit 3

Types of Number

Even Numbers

An Even Number is a number that is exactly divisible by 2 (a number in the 2 times tables).

Note: Even numbers end in 2, 4, 6, 8, 0. The first few even numbers are 2, 4, 6, 8, 10, 12, 14, …

Odd Numbers

An Odd Number is a number that is not an even number.

Note: Odd numbers end in 1, 3, 5, 7, 9. The first few odd numbers are 1, 3, 5, 7, 9, 11, 13, …

Square Numbers

You can get a Square Number by multiplying any whole number (integer) by itself

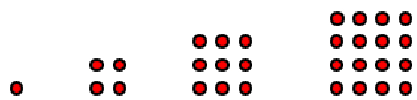
So: The first square number is 1, because 1 x 1 = 1.

The second square number is 4, because 2 x 2 = 4, and so on…

The first ten square numbers are: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100

Note:

You can also get all the square numbers by counting the dots in square patters:



Cube Numbers

You can get a Cube Number by multiplying any whole number (integer) by itself and then by itself again.

So: The first cube number is 1, because 1 x 1 x 1 = 1.

The second cube number is 8, because 2 x 2 x 2 = 8, and so on…

The first five cube numbers are: 1, 8, 27, 64, 125.

Prime Numbers

A Prime Number is a number that is only divisible by itself and 1; a prime number has exactly 2 factors.

For example: 7 is a prime number as it has two factors (1 and 7),

21 is NOT a prime number as it has four factors (1, 3, 7 and 21)

Note: 1 is **NOT** a prime number, as it only has one factor (1)

2 is the only **even** prime number as it has two factors (1 and 2)

The first ten prime numbers are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29.

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Unit 3

Factors

The Factors of a number are all the whole numbers (integers) that divide into your number exactly (there must not be a remainder).

For example: The factors of 12 are: 1, 2, 3, 4, 6 and 12, the factors of 55 are: 1, 5, 11, and 55

Note: 1 is a factor of all numbers, and so is the number itself.

Multiples

The Multiples of a number are all the numbers in the number's times table.

For example: The multiples of 2 are all the numbers in the 2 times table (2, 4, 6, 8, 10, …), the first three multiples of 6 are 6, 12, 18.

Reciprocals

To find the reciprocal of a whole number, turn it into a fraction by putting 1 over the number.

For example: The reciprocal of 7 would be $\frac{1}{7}$.

The reciprocal of 35 would be $\frac{1}{35}$.

To find the reciprocal of a fraction, flip the fraction upside down.

For example: The reciprocal of $\frac{3}{4}$ would be $\frac{4}{3}$.

The reciprocal of $\frac{1}{5}$ would be $\frac{5}{1} = 5$.

Index Form / Indices

Indices are just another word for "power".

The base → 5^4 ← The index or power

For example:

$3 \times 3 \times 3 \times 3 \times 3$ can be written as 3^5.

Five 3's multiplied together, so the index number is 5

2^4 means $2 \times 2 \times 2 \times 2$

Index number is 4, so four 2's multiplied together

For example: Calculate the value of 2^4.

$$\begin{aligned} 2^4 &= 2 \times 2 \times 2 \times 2 \\ &= 4 \times 2 \times 2 \\ &= 8 \times 2 \\ &= 16 \end{aligned}$$

For example: Calculate the value of $10^4 \times 2^3$.

$$\begin{aligned} 10^4 &= 10 \times 10 \times 10 \times 10 \\ &= 100 \times 10 \times 10 \\ &= 1000 \times 10 \\ &= 10000 \end{aligned}$$

$$\begin{aligned} 2^3 &= 2 \times 2 \times 2 \\ &= 4 \times 2 \\ &= 8 \end{aligned}$$

Work each one out separately, then multiply the answers

So, $10^5 \times 2^3 = 10000 \times 8 = 80000$

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Unit 3

Prime Factors

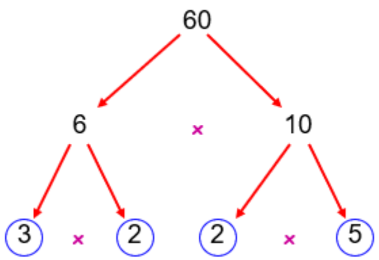
Prime Factors

Any positive integer can be written as a product of its prime factors.

Now, that may sound complicated, but all it means is that you can break up any number into a multiplication of prime numbers, and it's really easy to do with Factor Trees!

Don't Forget: 1 is NOT a prime number, so will NEVER be in your factor tree

Example: Express 60 as a product of its prime factors



3 x 2 x 2 x 5 = 60

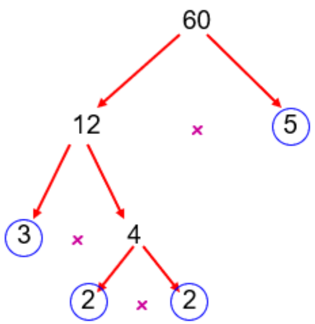
You can break the number up however you like:

6 x 10 or 12 x 5

Continue breaking up each new number into a multiplication

Stop when you reach a Prime Number and put a circle around it

Check your answer by multiplying all the numbers together



3 x 2 x 2 x 5 = 60

Note: Even though we started a different way, we still ended up with the same answer.

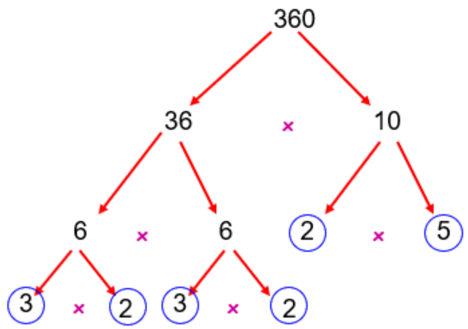
Try writing your answer starting with the smallest numbers:

$60 = 2 \times 2 \times 3 \times 5$

Then write the answer using indices:

$60 = 2^2 \times 3 \times 5$

e.g. Express 360 as a product of its prime factors



3 x 2 x 3 x 2 x 2 x 5 = 360

$360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5$

$360 = 2^3 \times 3^2 \times 5$

You can break the number up however you like. 36 x 10 is just easy to spot

Continue breaking up each new number into a multiplication

Stop when you reach a Prime Number and put a circle around it

Check your answer by multiplying all the numbers together

Write the numbers in order

If you can, use indices

Or you can try this 'ladder' method:

On the left:
Start with the given number.
All the other numbers are answers to the division from the right.
E.g. 60 ÷ 2 = 30
30 ÷ 2 = 15

60	2
30	2
15	3
5	5
1	

On the right:
All the numbers are **prime numbers** that go into the numbers on the left.
You divide by that prime number and write the answer on the left.
Continue this until you get to 1.

Check your answer:
Multiply together the numbers on the right:

2 x 2 x 2 x 5 = 60

Mathematics

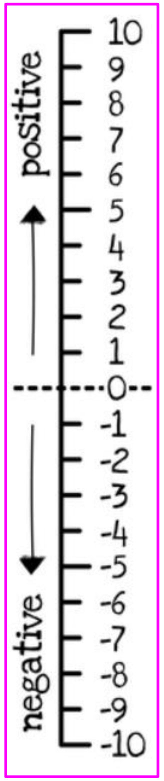
Foundation

Unit 4

Negative Numbers

Negative numbers can be represented on a number line.
You will notice that -3 is higher than -7.
It is possible to have negative temperatures when it is very cold (-3°C).

Number Line



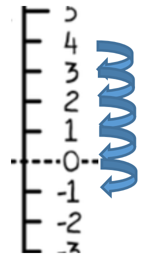
Temperature

Example: Complete the table below which shows the change in the midday temperatures on two successive days at four locations. The first row has been done for you.

Location	Temperature at midday on the first day (°C)	Change (°C)	Temperature on midday on the following day (°C)
Holyhead	-2	Up 3	1
Paris	4	Down 5	-1
Helsinki	-5	Down 2	-7
Glasgow	-1	Up 1	0

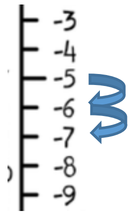
The temperature in Paris starts on 4 and ends up on -1.

Looking at the number line, to get from 4 to -1 we need to go down 5



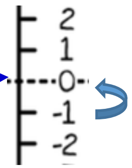
The temperature in Helsinki starts on -5 and goes down 2.

Looking at the number line, starting at -5 and going down 2 takes us to -7



The temperature in Glasgow goes up 1 and ends on 0.

What number on the number line do I need to start on so that I get to 0 by going up 1? -1



Ordering Directed Numbers

Think of a number line, which number would be further down the number line? Which number would be higher up?

Example 1: Put the following in ascending order

Smallest to biggest

12, 0, 23, -21, -17, -3

-21, -17, -3, 0, 12, 23

Example 2: Put the following in descending order

Biggest to smallest

-97, 85, 51, 2, -6, -47

85, 51, 2, -6, -47, -97

Mathematics

Foundation

Unit 4

Addition and Subtraction of Negative Numbers

For addition we move up the number line.

For subtraction we move down the number line.

When we have two signs (+ or -) immediately next to each other, we change the 2 signs into 1 using the following rules

$+\ + \rightarrow +$

$-\ - \rightarrow +$

$+\ - \rightarrow -$

$-\ + \rightarrow -$

The signs ARE NOT touching so we do not need to change them

Example 1: $-5 + 3 = -2$ Think of a number line, we start at -5 and move up 3 places

Example 2: $2 - 6 = -4$ Think of a number line, we start at 2 and move down 6 places

The signs ARE touching so we need to change them

Example 3: $4 + - 3 \rightarrow 4 \overset{-}{+ -} 3 \rightarrow 4 - 3 = 1$

Example 4: $-8 - -6 \rightarrow -8 \overset{+}{- -} 6 \rightarrow -8 + 6 = -2$

These **two** signs ARE touching so we need to change them

Think of a number line to help you work the answers out

Multiplying and Dividing Negative Numbers

Use the following rules:

$+$	$\times$	$+$	$\rightarrow$	$+$
$-$	$\times$	$-$	$\rightarrow$	$+$
$+$	$\times$	$-$	$\rightarrow$	$-$
$-$	$\times$	$+$	$\rightarrow$	$-$

$+$	$\div$	$+$	$\rightarrow$	$+$
$-$	$\div$	$-$	$\rightarrow$	$+$
$+$	$\div$	$-$	$\rightarrow$	$-$
$-$	$\div$	$+$	$\rightarrow$	$-$

Method: Multiply or divide the numbers first, then look at the signs using the rules

Example 1: -7×-3

Multiply the numbers: $7 \times 3 = 21$

Look at the signs: $- \times - = +$

(A negative x a negative = a positive)

So, $-7 \times -3 = 21$

If there is no sign it is a $+$

Example 2: $18 \div -6$

Divide the numbers: $18 \div 6 = 3$

Look at the signs: $+ \div - = -$

(A positive ÷ a negative = a negative)

So, $18 \div -6 = -3$

Mathematics

Foundation

Unit 5

Fractions

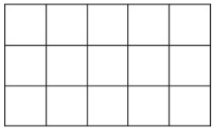
A **fraction** is part of a whole, made up of a numerator and a denominator $\frac{2}{3}$ ← Numerator ← Denominator

An **improper fraction** is a fraction where the numerator is greater than the denominator $\frac{18}{7}$ ← Numerator greater than denominator

A **mixed number** is a number made up of a whole number and a fraction. Whole number → $6\frac{4}{9}$ ← Fraction

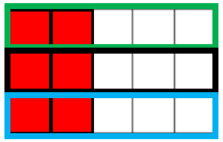
Shading a Fraction of a Shape

Example 1: Shade $\frac{2}{5}$ of this shape



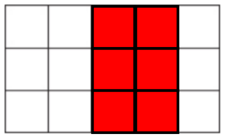
Option 1:

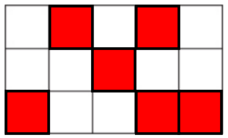
$\frac{2}{5}$ means 2 squares are shaded out of every 5 squares. We can group the shape into groups of 5 squares and shade 2 squares in each group.



Option 2:

Count the total number of squares in the shape – 15
Find $\frac{2}{5}$ of this number. (Remember, divide by the bottom, times by the top). $15 \div 5 \times 2 = 6$.
This is how many squares to colour in.
Does it matter what squares you colour?
No, any six squares would be fine.



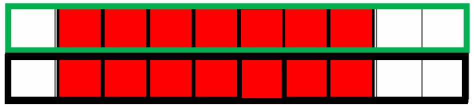


Shading a Fraction of a Shape – with percentages

Example 2: Shade 70% of this shape

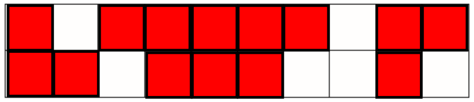
Option 1:

Write 70% as a fraction.
70% = $\frac{70}{100} = \frac{7}{10}$. This means we shade 7 squares out of every 10 squares.



Option 2:

Write 70% as a fraction. 70% = $\frac{70}{100} = \frac{7}{10}$.
Count the total number of squares in the shape – 20 squares.
Find $\frac{7}{10}$ of this number. (Remember, divide by the bottom, times by the top).



$20 \div 10 \times 7 = 14$.
This is how many squares to colour in.

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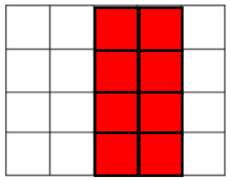
Unit 5

What Fraction is Shaded?



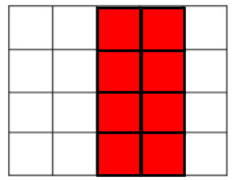
Example 1: What fraction of this shape is shaded? Write your answer in its simplest form.

8 squares are shaded

20 squares altogether

$$\frac{8}{20} = \frac{2}{5}$$

Simplify your answer



Example 2: What percentage of this shape is shaded?

Write your answer as a fraction first. Then change it to a percentage by putting it over 100.

$$\frac{8}{20} = \frac{40}{100} = 40\%$$

Equivalent Fractions

Equivalent fractions may look different, but they have the same value. To find an equivalent fraction the numerator and denominator must be **multiplied** or **divided** by the same number. Adding or subtracting the same number does not work as it does not remain in proportion.

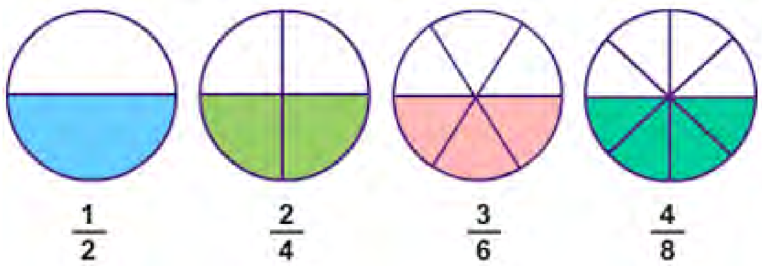
Example 1: $\frac{2}{4} = \frac{1}{2}$

Dividing the top and bottom of the fraction by 2 gives an equivalent fraction.

Example 2: $\frac{1}{4} = \frac{25}{100}$

Multiplying the top and bottom of the fraction by 25 gives an equivalent fraction.

Example of some equivalent fractions



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Unit 5

Simplifying Fractions

We can make fractions simpler, by dividing the numerator and the denominator by a common factor. (Questions may ask you to "simplify your answer").

Some fractions may simplify more than once, you need to keep simplifying until the fraction cannot be simplified any further.

Example 1: Simplify $\frac{10}{40}$

$$\frac{10}{40} \xrightarrow{\div 10} = \frac{1}{4}$$

Both 10 and 40 have been divided by 10 to make 1 and 4

Example 2: Simplify $\frac{24}{108}$

$$\frac{24}{108} \xrightarrow{\div 2} = \frac{12}{54} \xrightarrow{\div 2} = \frac{6}{27} \xrightarrow{\div 3} = \frac{2}{9}$$

Simplified in 3 steps

OR

$$\frac{24}{108} \xrightarrow{\div 12} = \frac{2}{9}$$

Simplified in 1 step

Same answer

Ordering Fractions

To be able to order fractions, they need to have the same denominator first.

Example: Put the following fractions in ascending order (smallest to biggest).

$$\frac{3}{4}, \frac{1}{2}, \frac{5}{6}, \frac{2}{3}$$

Step 1: Find the lowest common multiple of all the denominators

Lowest common multiple of 4, 2, 6, and 3 is 12

Step 2: Make equivalent fractions using the lowest common multiple as the denominator

$$\frac{3}{4} = \frac{9}{12}, \quad \frac{1}{2} = \frac{6}{12}, \quad \frac{5}{6} = \frac{10}{12}, \quad \frac{2}{3} = \frac{8}{12}$$

Step 3: Order the fractions, replace with original fractions

Smallest to biggest $\frac{6}{12}, \frac{8}{12}, \frac{9}{12}, \frac{10}{12}$

$$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}$$

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Foundation

Unit 5

Mixed Numbers to Improper Fractions

We can convert a mixed number to an improper fraction

E.g. $2\frac{1}{3}$ becomes $\frac{7}{3}$,

Example: Convert $2\frac{4}{7}$ to an improper fraction

Rule: $\dfrac{(\text{denominator x whole number}) + \text{numerator}}{\text{denominator}}$

$$2\frac{4}{7} = \frac{(7 \times 2) + 4}{7}$$

$$= \frac{14 + 4}{7}$$

$$= \frac{18}{7}$$

So, $2\frac{4}{7} = \frac{18}{7}$

Improper Fractions to Mixed Numbers

We can convert an improper fraction to a mixed number.

E.g. $\frac{7}{3}$ becomes $2\frac{1}{3}$.

Example: Convert $\frac{13}{5}$ to a mixed number

$\frac{13}{5}$ means 13 ÷5.

How many 5's are in 13? 2 (this becomes the whole number of the mixed number)

What is the remainder? 3 (this becomes the numerator of the fraction part of our mixed number)

How many 5's are in 13

So, $\frac{13}{5} = 2\frac{3}{5}$

Remainder

Denominator stays the same

Mathematics

Foundation

Unit 5

Finding a Fraction of a Quantity

To find a fraction of a quantity we use the rule: "Divide by the bottom, times by the top"

This means we divide the quantity by the denominator, then multiply the answer by the numerator.

Example 1 - Non-Calculator: Calculate $\frac{3}{4}$ of 20

Divide by the bottom: $20 \div 4 = 5$

Multiply by the top: $5 \times 3 = 15$

So, $\frac{3}{4}$ of 20 is 15

Make sure to write down your workings, either like above, or in one step $20 \div 4 \times 3 = 15$.

Example 2 - Calculator: Calculate $\frac{5}{7}$ of 17.5

You can type this straight into a calculator

$17.5 \div 7 \times 5 = 12.5$

So, $\frac{5}{7}$ of 17.5 is 12.5

Example 3: Sam comes from a large family. He has 80 relatives altogether, who live in Canada, Japan, and Wales. $\frac{1}{5}$ of his relatives live in Canada. $\frac{3}{8}$ of his relatives live in Japan. The rest of his relatives live in Wales. How many relatives live in Wales?

Work out how many relatives live in Canada:	$\frac{1}{5}$ of 80	$80 \div 5 = 16$		16 relatives live in Canada
Work out how many relatives live in Japan:	$\frac{3}{8}$ of 80	$80 \div 8 = 10$	$10 \times 3 = 30$	30 relatives live in Japan
Work out how many relatives are left:	$80 - 16 - 30 = 34$			34 relatives live in Wales

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Unit 5

Increasing/Decreasing by a Fraction

To find a fractional increase, first find the fraction of the quantity then add it to the original quantity.

To find a fractional decrease, first find the fraction of the quantity then subtract it from the original quantity.

Example 1: Increase £45 by $\frac{4}{9}$

Step 1: Find $\frac{4}{9}$ of £45:

45 ÷ 9 x 4 = £20

Step 2: This is a fractional increase question, so we add to the original quantity:

45 + 20 = £65

Example 2: Due to a bad summer, a farmer forecasts that her potato crop will be $\frac{2}{5}$ lower than the previous year.
She harvested 55 tonnes last year. What will it be this year?

Step 1: Find $\frac{2}{5}$ of 55 tonnes:

55 ÷ 5 x 2 = 22 tonnes

Step 2: The potato crop will be $\frac{2}{5}$ LOWER, it is a fractional decrease, so we subtract from the original quantity:

55 – 22 = 33 tonnes

Writing One Number as a Fraction of Another

Example 1: Write 36 as a fraction of 54, give your answer in its simplest form.

First write as a fraction $\frac{36}{54}$

Then simplify it $\frac{36}{54} = \frac{2}{3}$

Example 2: In a school of 280 pupils, 120 are boys. In its simplest form, what fraction of the pupils at the school are girls?

First work out how many girls there are: 280 – 120 = 160 girls

Write as a fraction

Number of girls → 160

Total number of pupils → 280

$\frac{160}{280}$

Then simplify it as much as possible

$\frac{160}{280} = \frac{16}{28} = \frac{4}{7}$

Simplify as much as possible

Mathematics

Foundation

Unit 5

Multiplying Fractions

To multiply fractions:

- Multiply both numerators;
- multiply both denominators;
- simplify the answer if possible, or cancel down within the question before multiplying

Example 1: $\frac{2}{5} \times \frac{5}{8}$ (simplifying the answer)

Numerators multiplied, 2 x 5 = 10

Denominators multiplied, 5 x 8 = 40

$$\frac{2}{5} \times \frac{5}{8} = \frac{10}{40} \overset{\div 10}{=} \frac{1}{4}$$

Example 2: $\frac{2}{5} \times \frac{5}{8}$ (simplifying within the question)

Cancel down any numerator and denominator. This means cancel down 2 and 8 by dividing by 2. Then cancel down both 5s by dividing by 5.

$$\frac{\cancel{2}^{1}}{\cancel{5}_{1}} \times \frac{\cancel{5}^{1}}{\cancel{8}_{4}} = \frac{1}{4}$$

Example 3: $\frac{4}{7} \times 3$ (multiplying a fraction by a whole number)

Write the 3 as a fraction by putting it over 1. Then multiply as above.

$$\frac{4}{7} \times \frac{3}{1} = \frac{12}{7} = 1\frac{5}{7}$$

Dividing Fractions

To divide fractions:

- Keep the first fraction the same;
- change the sign from a divide to a multiply;
- flip the second fraction upside down
- continue as you would for multiplying fractions

Example 1: $\frac{3}{4} \div \frac{5}{16}$ (dividing a fraction by a fraction)

$$\frac{3}{4} \div \frac{5}{16} = \frac{3}{4} \times \frac{16}{5}$$

$$= \frac{48}{20} \overset{\div 4}{=} \frac{12}{5} = 2\frac{2}{5}$$

Example 2: $\frac{9}{15} \div 3$ (dividing a fraction by a whole number)

$$\frac{9}{15} \div \frac{3}{1} = \frac{9}{15} \times \frac{1}{3}$$

Write the 3 as a fraction by putting it over 1. Then continue as above.

$$= \frac{9}{45} \overset{\div 3}{=} \frac{3}{15} \overset{\div 3}{=} \frac{1}{5}$$

Mathematics

Foundation

Unit 5

Adding and Subtracting Fractions

We can only add or subtract fractions with the same denominators.

Example 1: $\frac{2}{8} + \frac{3}{8} = \frac{5}{8}$

Add the top numbers

The bottom number stays the same

Example 2: $\frac{5}{9} - \frac{1}{9} = \frac{4}{9}$

Subtract the top numbers

The bottom number stays the same

When the denominators are different, we must change each fraction to have the same denominator first.

Example 3: $\frac{5}{6} + \frac{2}{3}$

The lowest common multiple of the denominators, 6 and 3, is 6.
This means we want both fractions to have a denominator of 6.

This fraction is already over 6 so we do not need to do anything to it $\frac{5}{6}$

$$\frac{2}{3} \overset{\times 2}{=} \frac{4}{6}$$

We now add $\frac{5}{6}$ and $\frac{4}{6}$ to obtain our answer.

Simplify the answer if possible

$$\frac{5}{6} + \frac{4}{6} = \frac{9}{6} = \frac{3}{2} = 1\frac{1}{2}$$

Example 4: $\frac{7}{15} - \frac{3}{10}$

The lowest common multiple of the denominators, 15 and 10, is 30.
This means we want both fractions to have a denominator of 30.

$$\frac{7}{15} \overset{\times 2}{=} \frac{14}{30} \qquad \frac{3}{10} \overset{\times 3}{=} \frac{9}{30}$$

We now subtract $\frac{9}{30}$ from $\frac{14}{30}$ to obtain our answer.

Simplify the answer if possible

$$\frac{14}{30} - \frac{9}{30} = \frac{5}{30} = \frac{1}{6}$$

Mathematics

Foundation

Unit 6

Decimals

Place Value and Ordering Decimals

Place value is the value given to a digit by its place in a number.

Ascending means smallest to biggest; descending means biggest to smallest

Decimal Place Value Table

Ten thousands	Thousands	Hundreds	Tens	Ones	.	Tenths	Hundredths	Thousandths	Ten thousandths

Example:

a) What is the value of the 9 in the number 10.609?
b) What is the value of the 7 in the number 234.75?

Using the place value table:

Ten thousands	Thousands	Hundreds	Tens	Ones	.	Tenths	Hundredths	Thousandths	Ten thousandths
			1	0	.	6	0	9	
		2	3	4	.	7	5		

a) 9 thousandths
b) 7 tenths

Ordering Decimal Numbers

Example: Put the following numbers in ascending order 43.85, 43.8, 43.856

Use the place value table to compare the numbers:

Ten thousands	Thousands	Hundreds	Tens	Ones	.	Tenths	Hundredths	Thousandths	Ten thousandths
			4	3	.	8	5	0	
			4	3	.	8	0	0	
			4	3	.	8	5	6	

Ascending means smallest to biggest, so we need the smallest number first.
All the whole numbers are of equal value so we need to start by looking at the decimal places. We can fill any gaps in with zeros to make comparing easier.
Looking at the tenths column, these digits are all the same. Looking at the hundredths column, the 0 is the smallest digit so 43.8 is the smallest number. The other 2 digits are the same so we look at the thousandths column. The zero is the smallest digit here so 43.85 is the next biggest number.

The order is: 43.8, 43.85, 43.856

Mathematics

Foundation

Unit 6

Adding and Subtracting Decimals

When we add or subtract decimals, we write the numbers out on top of each other, making sure we line up the decimal points.

Adding decimals 4.53 + 1.6

Line up the decimal points

Fill any gaps with zeros

```
  4.53      4.53      4.53
+ 1.60    + 1.60    + 1.60
  .  3     ₁.13      6₁.13
```

So 4.53 + 1.6 = **6.13**

Add the numbers, moving the decimal point in line to the answer

Example: 0.14 + 8 + 23.7

Step 1 and 2: Line up the decimal points, remember a whole number has an invisible decimal point after it, so 8 is the same as 8.0. Fill in any gaps with zeros

Decimal points lined up

```
   0 . 1 4
   8 . 0 0  +
 2 3 . 7 0
 _________

 _________
```

Step 3: Add the numbers, putting the decimal point in line in the answer

```
   0 . 1 4
   8 . 0 0  +
 2 3 . 7 0
 _________
 3 1 . 8 4
 _________
 1
```

Subtracting decimals 8.5 – 3.07

Fill any gaps with zeros

Line up the decimal points

```
  8 . ⁴5̸¹0     8 . ⁴5̸¹0     8 . ⁴5̸¹0
– 3 . 0 7    – 3 . 0 7    – 3 . 0 7
  .     3      . 4 3        5 . 4 3
```

So 8.5 – 3.07 = **5.43**

Subtract the numbers, moving the decimal point in line to the answer

Example: 47 – 19.43

Step 1 and 2: Line up the decimal points, remember a whole number has an invisible decimal point after it, so 47 is the same as 47.0. Fill in any gaps with zeros

Decimal points lined up

```
 4 7 . 0 0  -
 1 9 . 4 3
 _________

 _________
```

Step 3: Subtract the numbers, putting the decimal point in line in the answer

```
 ³4̸ ¹⁶7̸ . ⁹0̸ ¹0  -
 1  9  . 4  3
 ____________
 2  7  . 5  7
 ____________
```

Mathematics

Foundation

Unit 6

Multiplying Decimals

When we multiply decimals, we ignore the decimal point and just multiply the numbers, then we count how many decimal places (numbers after the decimal point) there are in the question and put the decimal point back in the answer making sure we have the same number of decimal places in our answer.

Multiplying Decimals 0.03 x 1.1

Ignore the decimal points and multiply the numbers

3 x 11 = 33

(0.03 x 1.1)

So: 0.03 x 11 = 0.033

Count how many decimal places there are in the question (3 decimal places altogether)

Count backwards from the end of the answer the same number of decimal places, put in the decimal point

Example 1: 8 x 0.5

Step 1: Ignore the decimal points and multiply the numbers 8 x 5

8 x 5 = 40

Step 2: Count the number of decimal places in the question

8 x 0.5 One decimal place

Step 3: Count backwards from the end of the answer the same number of decimal places, put in the decimal point.

4 . 0

So, 8 x 0.5 = 4 (which is the same as 4.0)

Example 2: 2.6 x 2.3

Step 1: Ignore the decimal points and multiply the numbers 26 x 23 (This is a long multiplication, look back at unit 1 if you are unsure how to do it)

$$
\begin{array}{r@{\,}l}
2\ 6 & \times \\
2\ 3 & \\
\hline
7\ 8 & \\
5\ 2\ 0 & + \\
\hline
5\ 9\ 8 & \\
\hline
\end{array}
$$

Step 2: Count the number of decimal places in the question

2.6 x 2.3 Two decimal places

Step 3: Count backwards from the end of the answer the same number of decimal places, put in the decimal point.

5 . 9 8

So, 2.6 x 2.3 = 5.98

Mathematics

Foundation

Unit 6

Dividing Decimals

When we divide a decimal by a whole number (e.g. 18.6 ÷ 3) we can set it up like a normal dividing question using the bus stop method, moving the decimal point in line up into the answer.

When we divide a whole number by a decimal (e.g. 25 ÷ 0.5), or a decimal by a decimal (e.g. 18.6 ÷ 0.03) we need to make the number we are dividing by into a whole number first, then divide in the usual way.

Dividing Decimals by a whole number 2.52 ÷ 4

Set up using the bus stop method, divide as usual, the decimal point moves in line up into the answer

$$\begin{array}{r} 0\,.\,6\,3 \\ 4\overline{\big)\,2\,.\,{}^{2}5\,{}^{1}2} \end{array}$$

Example: 12.84 ÷ 6

Set up using the bus stop method, divide as usual, the decimal point moves in line up into the answer

$$\begin{array}{r} 0\,2\,.\,1\,4 \\ 6\overline{\big)\,1\,{}^{1}2\,.\,8\,{}^{2}4} \end{array}$$

Dividing by a Decimal 4.06 ÷ 0.2

We do not like to divide by a decimal, so we make it a whole number first. We can do this by either multiplying by 10, 100, 100 …

Remember, whatever we multiply the one number by we must also multiply the other by as well.

Also multiply by 10, must do the same to both sides

4.06 x 10 = 40.6 0.2 x 10 = 2

Multiply by 10 to make 0.2 a whole number

We are now working out 40.6 ÷ 2

$$\begin{array}{r} 2\,0\,.\,3 \\ 2\overline{\big)\,4\,0\,.\,6} \end{array}$$

So, 4.06 ÷ 0.2 = 20.3

Answer to 4.06 ÷ 0.2 is the same as the answer to 40.6 ÷ 2

Example: 45 ÷ 0.04

Step 1: Multiply both sides by 100 to make 0.04 a whole number

45 x 100 = 4500 0.04 x 100 = 4

We are now working out 4500 ÷ 4

Step 2: Set up using the bus stop method, divide as usual

$$\begin{array}{r} 1\,1\,2\,5 \\ 4\overline{\big)\,4\,5\,{}^{1}0\,{}^{2}0} \end{array}$$

So, 45 ÷ 0.04 = 1125

Mathematics

Foundation

Unit 7

Round to an Appropriate Degree of Accuracy

There are lots of degrees of accuracy you will need to know how to round to, but the way to work out any rounding question is always the same:

Step 1: Circle the last digit you need – what I will call the Key Digit

Step 2: Look at the unwanted digit to the right to it – if it is 5 or above add one on to your Key Digit, if it is less than five, leave your Key Digit alone.

Step 3: Be very careful of the dreaded number 9…

Rounding to Nearest Whole Number, 10, 100, 1000 etc

Remember: the size of your rounded number should be a similar size to the number in the question, and you must use zeros to help you with this.

Example 1

Round 3.825 to the nearest whole number

3 . 8 2 5

1. Our Key Digit is always the degree of accuracy the question asks for, which in this case is whole numbers, so we need the 3.

2. The unwanted digit to the right of it is 8, which is more than 5, so we add one to our Key Digit.

3. So, to the nearest whole number, our answer is:

4

Example 2

Round 4,365,901 to the nearest thousand

4 3 6 5 9 0 1

1. We want the nearest thousand, so our Key Digit must be the number that represents the thousands which is the 5

2. The unwanted digit to the right of it is 9, which is more than 5, so we add one to our Key Digit.

3. So, to the nearest whole number, our answer is:

4,365,000

Example 3

Round 3,999 to the nearest ten

3 9 9 9

1. We want the nearest ten, so the Key Digit must be the 9 in the tens column

2. The unwanted digit to the right of it is a 9, so we add one on, but we then need to add one on the next 9, and then the 3.

3. So, to the nearest ten, our answer is:

4,000

Mathematics

Foundation

Unit 7

Rounding to Decimal Places

You will be asked to round to a given number of decimal places, this can be written as d.p.

E.g. 5.95783… rounded to 2 d.p. is 5.96

Remember, if the question asks for two decimal places, you must give two, no more, no less.

Example 1

Round 5.639 to 1dp

5 . 6 3 9

1. We start by putting a ring around our Key Digit. The question has asked for 1 decimal place, so our key digit is the 6, as it occupies the 1st decimal place

2. Next we look at the digit to the right to it – the unwanted number 3. It is less than 5, so we leave the key digit alone.

3. So, to one decimal place, our answer is:

5.6

Example 2

Round 12.0482 to 2dp

1 2 . 0 4 8 2

1. This time the Key Digit is in the 2nd decimal place, which makes it the 4

2. The unwanted digit to the right of it is an 8, which is more than 5, so we must add one onto our Key Digit

3. So, to two decimal places, our answer is:

12.05

Example 3

Round 25.72037 to 3dp

2 5 . 7 2 0 3 7

1. This time the Key Digit is in the 3rd decimal place, which makes it the 0

2. The unwanted digit to the right of it is 3, which is less than 5, so just leave our Key Digit alone

3. So, to three decimal places, our answer is:

25.720

Be careful: The answer is not 25.72, as we must have the 3 decimal places.

Example 4

Round 3.7952 to 2dp

3 . 7 9 5 2

1. This time the Key Digit is in the 2nd decimal place, which makes it the 9

2. The unwanted digit to the right of it is a 5, which is 5 or above, so we must add one onto our Key Digit

But: if we add one to our key digit, we get 10. So, we must add one to the next digit as well, which is the 7

3. So, to two decimal places, our answer is:

3.80

Mathematics

Foundation

Unit 7

Rounding to Significant Figures

You may be asked to round to a given number of significant figures, this can be written as s.f.

E.g. 59,578 rounded to 1 s.f. is 60,000

Note: The first significant figure is always the first non-zero digit you come across.

Remember: the size of your rounded number should be a similar size to the question, and you must use zeros to help you with this.

Example 1

Round 28.53 to 1 sig fig

2 8 . 5 3

1. The Key Digit is the first significant figure, which must be the 2, as it is the first non-zero number

2. Look to the number to the right, which is an 8, we add one on.

3. So, keeping the size of the answer the same as the question with a zero, to 1 sig fig the answer must be:

30

Example 2

Round 5,322 to 1 sig fig

5 3 2 2

1. The Key Digit is the first significant figure, which must be the 2, as it is the first non-zero number

2. The unwanted digit to the right of it is 3, which is less than 5, so we leave our Key Digit alone

3. Again using zeros to help us, to one sig fig, our answer is:

5000

Example 3

Round 0.027 to 1 sig fig

0 . 0 2 7

1. Our first significant figure is the first non-zero number, which means it's the 2

2. The unwanted digit to the right of it is 7, so we add one to our Key Digit.

3. No need for extra zeros here, so to the 1 significant figure our answer is:

0.03

Mathematics

Foundation

Unit 7

Estimating

When we estimate we round each number to one significant figure to make the calculations easier to do.

E.g. 231×8.9

$200 \times 9 = 1800$

So, the actual answer is approximately 1800.

Estimate the value of $\frac{32.5 \times 38.3}{190}$

Step 1: First round each value to 1 significant figure: $\rightarrow \frac{30 \times 40}{200}$

Step 2: Apply the rules of BIDMAS/BODMAS to calculate the answer:

$$\rightarrow \frac{30 \times 40}{200} \rightarrow \frac{1200}{200} \rightarrow \frac{12\cancel{00}}{2\cancel{00}} \rightarrow \frac{12}{2} \rightarrow 6$$

The actual answer on the calculator is close: 6.22375...

We can say that: $\frac{32.5 \times 38.3}{190} \approx 6$

approximately equal to

Example:

Estimate the value of: $\frac{64 \times 290}{2001}$

$$\frac{64 \times 290}{2001}$$

Rounded to 1 s.f. $\approx \frac{60 \times 300}{2000}$

$$\approx \frac{18000}{2000}$$

$$\approx \frac{18\cancel{000}}{2\cancel{000}}$$

$$\approx \frac{18}{2}$$

$$\approx 9$$

Mathematics

Foundation

Unit 8

Percentages

A percentage is just a fraction whose denominator (bottom) is 100. So, if we say "32%", what we mean is $\frac{32}{100}$ or 32 out of 100.

Percentage of an Amount – Non-Calculator

Method We can calculate all percentages by first calculating some of these:

Example: You have £320. Find (a) 15%, (b) 63%, (c) 17.5%

Start by writing down the percentages that you know which might help:

To find 10% → Divide by 10 → 320 ÷ 10 = 32 → 10% = £32

To find 1% → Divide by 100 → 320 ÷ 100 = 3.2 → 1% = £3.20

To find 50% → Divide by 2 → 320 ÷ 2 = 160 → 50% = £160

To find 20% → Double 10% → 32 x 2 = 64 → 20% = £64

To find 5% → Half 10% → 32 ÷ 2 = 16 → 5% = £16

To find 2.5% → Half 5% → 16 ÷ 2 = 8 → 2.5% = £8

You can build up your answers with a bit of simple addition.

(a) 15%

15% = 10% + 5%
= £32 + £16
= £48

(b) 63%

63% = 50% + 10% + 1% + 1% + 1%
= £160 + £32 + £3.20 + £3.20 + £3.20
= £201.60

(c) 17.5%

17.5% = 10% + 5% + 2.5%
= £32 + £16 + £8
= £56

Percentage of an Amount - Calculator

Finding a percentage of an amount using a calculator can be done in one easy step.

Example: Find 23% of 135g (23% percent can be written as 23 ÷ 100, or $\frac{23}{100}$)

Type into the calculator: $23 \div 100 \times 135 =$

Make sure you write the workings down as well as the answer

$$23 \div 100 \times 135 = 31.05g$$

One Number as a Percentage of Another

Example Non-Calculator:

Write 19 as a percentage of 25?

Step 1: Write as a fraction and multiply it by 100

$$\frac{19}{25} \times 100$$

Step 2: Multiply (look back at Unit 5 to recall how to multiply a fraction by a whole number)

$$\frac{19}{\cancel{25}_{1}} \times \frac{\cancel{100}^{4}}{1} = 76\%$$

(19 is 76% of 25)

Example Calculator:

Write 256 as a percentage of 780?

Step 1: Type into the calculator

256 ÷ 780 x 100 =

Make sure you write the workings down as well as the answer.

256 ÷ 780 x 100 = 32.82% (2 dp)

(256 is 32.82% of 780)

Mathematics

Foundation

Unit 8

Percentage Increase

If we increase an amount it means it will get bigger.
So to increase an amount by 10%, we find 10% of the amount and add it on.

Example 1 - Non-Calculator:

Increase £250 by 15%

Step 1: Find 15% of 250

10% = 25

5% = 12.5

15% = 25 + 12.5

= £37.50

Step 2: Increase means to add on

250 + 37.50 = £287.50

Example 2 - Calculator:

Increase £250 by 15%

Step 1: Type into the calculator

15 ÷ 100 x 135 =

Make sure you write the workings down as well as the answer.

15 ÷ 100 x 135 = £37.50

Step 2: Increase means to add on

250 + 37.50 = £287.50

Both methods give the same answer.

Percentage Decrease

If we decrease an amount it means it will get smaller.
So to decrease an amount by 25%, we find 25% of the amount and take it away.

Example 1 - Non-Calculator:

Decrease 350g by 21%

Step 1: Find 21% of 350

10% = 35

1% = 3.5

21% = 35 + 35 + 3.5

= 73.5g

Step 2: Decrease means to take away

350 - 73.5 = 276.5g

Example 2 - Calculator:

Decrease 350g by 21%

Step 1: Type into the calculator

21 ÷ 100 x 350 =

Make sure you write the workings down as well as the answer.

21 ÷ 100 x 350 = 73.5g

Step 2: Decrease means to add on

350 - 73.5 = 276.5g

Both methods give the same answer.

Mathematics

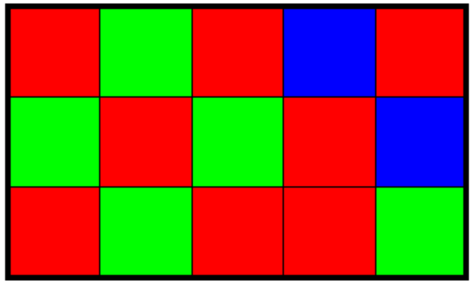
Foundation

Unit 9

Ratio and Proportion

Writing Ratios

Ratios require the use of a colon **:**



The ratio of red squares to green squares is:

8 : 5

Because for every 8 red squares, there are 5 green:

The ratio of green squares to red squares is:

5 : 8

The ratio of blue squares to red squares is:

2 : 8

Simplifying Ratios

Method Just like with fractions, whatever you multiply/divide one side by, make sure you do the exact same to the other side. Keep dividing until each side has no common factors

Example 1: Simplify 14 : 21

We are looking for factors common to both sides, let's try 7.

Divide both sides by 7

$$\div 7 \quad 14 : 21 \quad \div 7$$
$$2 : 3$$

Check: Are there are any other common factors to simplify it further? No, we have simplified it as far as possible.

Example 2: Simplify 60 : 45

We are looking for factors common to both sides, let's try 5.

Divide both sides by 5

$$\div 5 \quad 60 : 45 \quad \div 5$$
$$12 : 9$$

Check: Are there are any other common factors to simplify it further? Yes, 3 is a common factor to both sides.

Divide both sides by 3

$$\div 3 \quad 12 : 9 \quad \div 3$$
$$4 : 3$$

Check: Are there are any other common factors to simplify it further? No, we have simplified it as far as possible.

Note: For example 2 we could have divided both sides by 15 to start, which would have given us our answer of 4 : 3 in one step. It does not matter which way you choose, just make sure you simplify as much as possible.

Mathematics

Foundation

Unit 9

Proportional Division

Remember: Whatever you multiply/divide one side by, do the same to the other.

Example 1:

Emma is making a cake. On the packet it says that the ingredients must be mixed in the following ratios:

Flour (g)	:	Butter (g)	:	Eggs	:	Sugar (g)
400	:	220	:	3	:	25

(a) If my Emma has 1000g of flour, how much butter does she need?

(b) If she has 2 eggs, how much sugar does she need?

Always set these sort of questions out the same way - write the original ratios on the top, write the new amount you know on the bottom, and ask yourself: "what do I need to do to get from my original amount to my new amount?"

(a) This is what we've got:

flour butter

$$400 : 220$$
$$\times 2.5 \qquad \times 2.5$$
$$1000 : ?$$

$1000 \div 400 = 2.5$

How do I get from 400 to 1000? I multiply by 2.5, do the same to the butter.

220 x 2.5 = 550g

(b) This is what we've got:

eggs sugar

$$3 : 25$$
$$\times 2/3 \qquad \times 2/3$$
$$2 : ?$$

$2 \div 3 = 2/3$

How do I get from 3 to 2? I multiply by $2/3$, do the same to the sugar.

$$25 \times 2/3 = 16\frac{2}{3}\text{g}$$

Example 2:

Box A has 8 fish fingers costing £1.40.
Box B has 20 fish fingers costing £3.40.
Which box is the better value?

First work out what it costs for one fish finger for each box.

Box A – 8 fish fingers so divide the cost by 8

$$A = \frac{£1.40}{8}$$
$$= £0.175$$

Box B – 20 fish fingers so divide the cost by 20

$$B = \frac{£3.40}{20}$$
$$= £0.17$$

Therefore, Box B is better value as the cost for one fish finger is less.

Mathematics

Foundation

Unit 9

Sharing in a Given Ratio

Method for Sharing Ratios

Step 1: Add up the total number of parts you are sharing between

Step 2: Work out how much one part gets

Step 3: Use this to work out how much everybody gets.

Example 1:

24 chocolates are to be shared between Mary and Jacob in the ratio 5 : 3. Work out how many chocolates each person gets.

Step 1: Mary gets 5 parts and Jacob gets 3 parts, so in total there are 8 parts.

Step 2: There are 24 pieces of chocolate all together, so one part is worth

$24 \div 8 = 3$ pieces

Step 3: Mary has 5 parts:

$5 \times 3 = 15$ pieces

Jacob has 3 parts:

$3 \times 3 = 9$ pieces

$\boxed{15 + 9 = 24}$

Example 2:

Share £845 in the ratio 8 : 3 : 2

Step 1: In total there are 13 parts (8 + 3 + 2)

Step 2: We have £845 to share, so one part is worth

$845 \div 13 = £65$

Step 3:

8 parts	$8 \times 65 = £520$
3 parts	$3 \times 65 = £195$
2 parts	$2 \times 65 = £130$

Check: 520 + 195 + 130 = £845

In this example you do not know the total amount.

Example 3:

Tom and Lisa share money in the ratio 8:3. Tom has £40, how much does Lisa have?

Tom gets 8 parts which is worth £40.

One part is worth

$40 \div 8 = £5$

Lisa has 3 parts:

$3 \times £5 = £15$

You may even be asked how much money was there altogether. In this example £40 + £15 = £55

Mathematics

Foundation

Unit 9

Ratio in Scale Drawings or Maps

For ratio problems involving scale drawings or maps, write the ratio as 'map: real life' and be careful with units. You will probably be required to convert between units.

Remember: Whatever you multiply/divide one side by, do the same to the other.

Example:

Kate and Ben planned a cycle ride using a 1:25 000 scale map. The route they planned measured approximately 80cm on the map.

a) Calculate how far they planned to cycle. Give your answer in km.
b) After the ride, Kate's watch showed they had travelled 24km. What was this measurement on the map in cm?

Always write the original ratios on the top with units

a) map : real life

1cm : 25 000 cm

80cm : ?

How do I get from 1cm to 80cm? Multiply by 80 so do the same to 25 000cm.

25 000 x 80 = 2 000 000cm

÷ by 100 to get into metres

2 000 000cm = 20 000m

÷ by 1000 to get into kilometres

20 000m = 20km

80cm on the map is equivalent to 20km in real life

b) map : real life

1cm : 25 000 cm

? : 24km

? : 2 400 000cm

Covert km into cm first (x 1000 and then x 100)

How do we get from 25 000 to 2 400 000?

2 400 000 ÷ 25 000 = 96, so multiply by 96

1 x 96 = 96cm

24km in real life is equivalent to 96cm on the map

Mathematics

Foundation

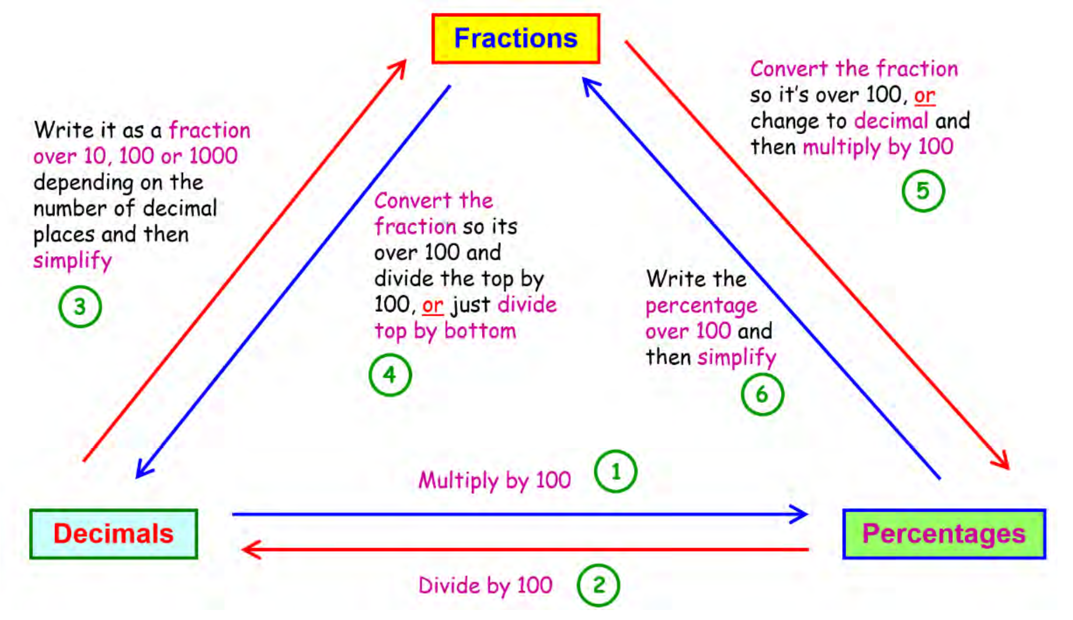
Unit 10

Fractions, Decimals and Percentages

Fractions, Decimals and Percentages are all closely related to each other, and you need to be comfortable changing between each of them.

You can use this diagram to help you.

Follow the arrows depending on what you need to change and follow the numbers for the examples.



Examples:

1 What is 0.364 as a percentage?

Just multiply by 100 and be careful with the decimal point!

0.364×100

$= 36.4\%$

2 Convert 8.3% into a decimal

Just divide by 100 and again be careful with the decimal point!

$8.3 \div 100$

$= 0.083$

3 Write 0.16 as a fraction

There are 2 decimal places, so write it over 100

$\frac{16}{100}$

Now carefully simplify

$\frac{16}{100} = \frac{8}{50} = \frac{4}{25}$

4 Write $\frac{13}{20}$ as a decimal

We need to change the bottom of the fraction to 100, remembering to do the same to the top

$\frac{13}{20} \overset{\times 5}{=} \frac{65}{100}$

Divide the top of your fraction by 100 and you have your answer!

$= 0.65$

5 Write $\frac{5}{8}$ as a percentage

It's not easy to change this fraction over 100, so we must divide 5 by 8

$5 \div 8$

Use any method, but I do this:

$= 8\overline{)5.000}$ giving 0.625

0.625 is the answer as a decimal, so we must multiply by 100

0.625×100

$= 62.5\%$

6 What is 12.5% as a fraction?

Start by writing the percentage over 100

$\frac{12.5}{100}$

We need to simplify, but the decimal point makes it hard. So why not multiply top and bottom by 2!

$\times 2$ $\frac{25}{200}$

Now we can simplify as normal to get the answer:

$\frac{25}{200} = \frac{5}{40} = \frac{1}{8}$

Mathematics

Foundation

Unit 10

Ordering Fractions, Decimals and Percentages

To order a mix of fractions, decimals, and percentages you need to first convert all the numbers to the same form, either fractions, decimals, or percentages.

Note: Ascending Order means smallest to largest.

Descending Order means largest to smallest.

Example:

Put the following in ascending order

56% $\frac{3}{4}$ 0.871 23% $\frac{6}{7}$

To order these, convert them all to decimals.

56%	$\frac{3}{4}$	0.871	23%	$\frac{6}{7}$
0.56	0.75	0.871	0.23	0.857...
2	3	5	1	4

Then write the correct order but as they were in the original question.

23% 56% $\frac{3}{4}$ $\frac{6}{7}$ 0.871

Here are some equivalent fractions, decimals, and percentages you should know.

F	D	P
$\frac{1}{100}$	0.01	1%
$\frac{1}{10}$	0.1	10%
$\frac{1}{5}$	0.2	20%
$\frac{1}{4}$	0.25	25%
$\frac{1}{2}$	0.5	50%
$\frac{3}{4}$	0.75	75%
$\frac{1}{3}$	$0.\dot{3}$	$33.\dot{3}\%$
$\frac{2}{3}$	$0.\dot{6}$	$66.\dot{6}\%$

Recurring Decimals

Some decimals **terminate**, which means the decimals do not recur, they just stop. For example, 0.75.

A **recurring decimal** exists when decimal numbers repeat forever.

Convert $\frac{8}{11}$ into a decimal using your calculator. A calculator displays this as $0.\dot{7}\dot{2}$ or 0.7272727272......

The digits 2 and 7 repeat infinitely. This is an example of a **recurring decimal**.

We can show this by writing dots above the 7 and the 2 (the numbers that recur).

If you had to convert into a recurring decimal without the calculator, you would need to use the bus shelter method

Write $\frac{5}{6}$ as a decimal

$$\begin{array}{r} 0.8333 \\ 6\,\overline{)\,5.0000} \end{array}$$

So, $\frac{5}{6} = 0.8\dot{3}$

Here are some more examples of recurring decimals:

$\frac{4}{9} = 0.\dot{4}$ This decimal is made up of an infinite number of repeating 4s.

$\frac{5}{6} = 0.8\dot{3}$ This decimal starts with an 8 and is followed by an infinite number of repeating 3s.

$\frac{2}{7} = 0.\dot{2}8571\dot{4}$ In this decimal, the six digits 285714 repeat an infinite number of times in the same order.

$\frac{9}{22} = 0.4\dot{0}\dot{9}$ This decimal starts with a 4. The two digits 09 then repeat an infinite number of times.

Mathematics

Foundation

Unit 11

Measure

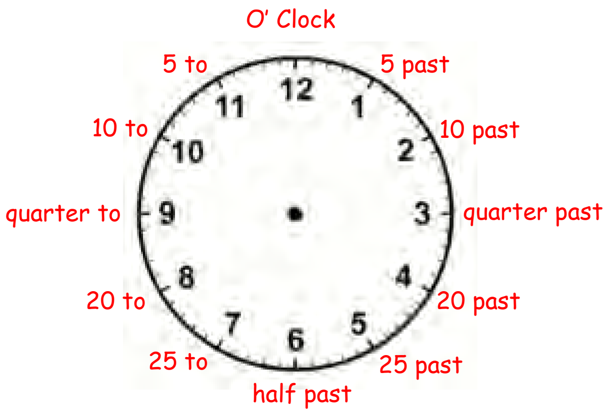
Time

There are two different types of time, analogue and digital.

An analogue clock or watch has moving hands that show you the time.

A digital clock or watch has numbers. Digital times can also be shown as 12-hour or 24-hour times.

Analogue and Digital, 12-hour and 24-hour



Small hand of clock (hour hand)

large hand of clock (minute hand)

11:20

Example 1:

3:45

The time is quarter to 4, the hour hand has gone past the 3 and nearly reached the 4, the minute hand is on the 9 (quarter to).

Example 2:

23:30

The time is half past 11 (p.m.), the hour hand has gone past the 11, the minute hand is on the 6 (half past).

Example 3:

9:10

The time is 10 past 9, the hour hand has just gone past the 9, the minute hand is on the 2 (10 past).

Example 4:

00:55

The time is 5 to 1 (a.m.), the hour hand has gone past the 12 and nearly reached the 1, the minute hand is on the 11 (5 to).

12-hour	24-hour
12 am	00:00
1am	01:00
2am	02:00
3am	03:00
4am	04:00
5am	05:00
6am	06:00
7am	07:00
8am	08:00
9am	09:00
10am	10:00
11am	11:00
12 pm Lunch time	Noon
1pm	13:00
2pm	14:00
3pm	15:00
4pm	16:00
5pm	17:00
6pm	18:00
7pm	19:00
8pm	20:00
9pm	21:00
10pm	22:00
11pm	23:00

Mathematics

Foundation

Unit 11

Metric units of **length**:

mm, cm, m, km

Metric units of **weight**:

g, kg, tonne

Metric units of **capacity**:

ml, cl, litre

Conversions:

1ml = 1cm^3

1 litre = 1000cm^3

1m^2 = 10000cm^2

Units

Converting length:

$\div 10$ $\div 100$ $\div 1000$

mm cm m km

$\times 10$ $\times 100$ $\times 1000$

Converting capacity:

$\div 1000$ $\div 100$

ml l cl

$\times 1000$ $\times 100$

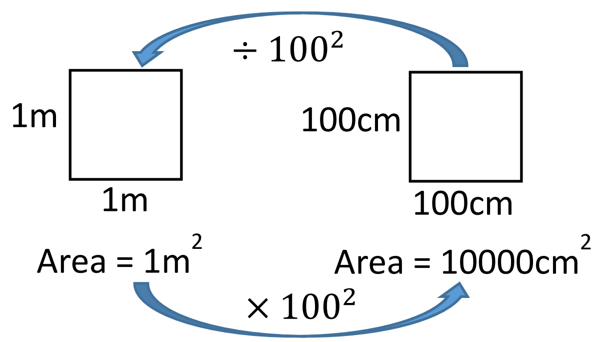
Converting mass:

$\div 1000$ $\div 1000$

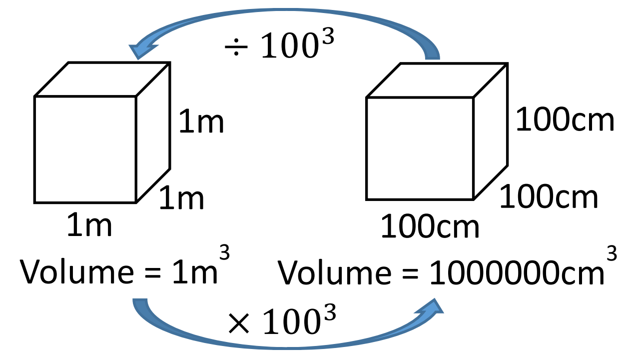
g kg tonne

$\times 1000$ $\times 1000$

Converting area:



Converting volume:



Metric- Imperial approximations

1 mile ≈ 1.6km

$\div 1.6$

km miles

$\times 1.6$

1kg≈ 2.2pounds

$\div 2.2$

pounds kg

$\times 2.2$

1 litre ≈ 1.75 pints

$\div 1.75$

pints litre

$\times 1.75$

Mathematics

Unit 11

Example 1: What units would we use to measure:

a) The length of a hand? cm

b) The distance from here to London? km

c) The weight of an apple? g

d) The weight of a man? kg

e) A spoonful of medicine? ml

f) A bucket of water? l

Example 2: To change from km to m $\times 1000$

How many metres are there in 5·07 kilometres?

$5.07 \times 1000 = 5070m$

Example 3: Circle the appropriate quantities for each measurement.

Weight of a bicycle	$7\ mg$	$7\ g$	$7\ kg$
Capacity of a juice carton	$1\ l$	$1\ ml$	$1\ cl$
Height of a man	$17\ m$	$1.7\ m$	$17\ cm$

Example 4:

(a) Change 600 mm to cm. To change from mm to cm $\div 10$

$600 \div 10 = 60cm$

(b) Change 2800 mm to m.

Change from mm to cm first $2800 \div 10 = 280cm$

Then change from cm to m $280 \div 100 = 2.8m$

(c) The hotel was 3 km from the port.

(i) How far is this in metres? To change from km to m $\times 1000$

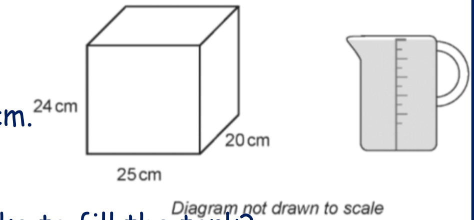
$3 \times 1000 = 3000m$

(ii) How far is this in miles? Give your answer correct to the nearest mile.

To change from km to $miles$ $\div 1.6$ $3 \div 1.6 = 1.875\ miles$

$= 2\ miles$ (to nearest mile)

Example 5: A jug holds one and a half litres of water when full. A tank has dimensions 25cm by 24cm by 20cm.



How many full jugs of water will it take to fill the tank?

Volume of tank $= 24 \times 25 \times 20$

$= 12000cm^3$

Jug holds 1.5 litres

(1 litre = $1000cm^3$)

$1.5 \times 1000 = 1500cm^3$

Number of full jugs needed to fill tank: $12000 \div 1500 = 8$

8 full jugs are needed to fill the tank.

Mathematics

Foundation

Unit 12

Interpret and Use Mathematical Information

There are:

- 60 seconds in one minute;
- 60 minutes in one hour;
- 24 hours in one day;

There are:

- 7 days in a week;
- 52 weeks in a year;
- 12 months in a year;
- 365 days in a year;
- 366 days in a leap year.

The 12 months of the year are:

January, February, March, April, May, June, July, August, September, October, November, and December

We can remember how many days are in each month with the rhyme:

"30 days have September, April June and November. All the rest have 31, except for February alone, which has 28 days each year, 29 days each leap year".

Tv Schedules

Example: A television channel needs to fit the following four programmes between its three "News" slots.

Programme	Time needed
Your Songs	30 minutes
Nature Trails	25 minutes
Theatre Review	20 minutes
The Comedy Slot	35 minutes

The title of each news bulletin tells you how long it lasts.

Complete the timetable to the left to show the order in which the four programmes can be broadcast.

We have from 11:10 a.m. until 11:55 a.m. to slot some programmes in. This is 45 minutes. There are only 2 programmes that add to 45 minutes, it does not matter which is first.

15-minute news update starts at 11:55 a.m. and finishes at 12:10 p.m.

Time	Programme
11:00 a.m.	The 10-minute News Report
11:10 a.m.	Theatre Review
11:30 a.m.	Nature Trails
11:55 a.m.	Your 15-minute News Update
12:10 p.m.	Your Songs
12:40 p.m.	The Comedy Slot
1:15 p.m.	The 10-minute News Report

We have from 12:10 a.m. until 1:15 p.m. to slot some programmes in. This is 65 minutes. There are only 2 programmes that add to 65 minutes, it does not matter which is first.

Mathematics

Foundation

Unit 15

Timetables and Time

Example 1:

The following tables are parts of train timetables between Reading and London and between London and Birmingham.

Reading	09:55	10:03	10:10	10:38	11:26
London	10:25	10:44	10:49	11:17	11:57

London	15:03	15:23	15:43	15:54	16:50
Birmingham	16:27	16:45	17:08	17:17	18:11

Andrew catches the 10:38 train from Reading to London.
How long should the journey take?

10:38 → 11:17

10:38 → 11 = 22 minutes

11 → 11:17 = 17 minutes

22 + 17 = 39 minutes

Example 2:

When it is 19:40 in Cardiff, it is 23:40 in Dubai.

19:40 → 23:40
+4hrs

(i) What time is it in Dubai when it is 13:30 in Cardiff?
Circle your answer.

15:30 10:30 09:30 17:30 19:30

13:30 → 17:30

+4hrs

(ii) What time is it in Cardiff when it is 02:10 in Dubai?
Circle your answer.

20:10 06:10 22:10 10:10 00:10

Cardiff → Dubai +4hrs

Dubai → Cardiff -4hrs

02:10 → 22:10

Mathematics

Foundation

Unit 12

Distance tables

The chart below shows the road distances between some towns and cities.
The distances are given in miles.

Abergavenny			
18	**Newport**		
45	53	**Gloucester**	
50	32	36	**Bristol**

Look down the column from Abergavenny and across the row from Bristol where they meet is the answer

Wyn lives in Abergavenny and works in Bristol.

(a) Use the chart to find the road distance from Abergavenny to Bristol.

50 miles

Wyn works in Bristol for 5 days each week.
Each day, he drives to and from work using the route shown on the map.

Diagram not drawn to scale

How many miles, in total, does he travel to and from work each week?

Wyn travels from Abergavenny to Newport and then Newport to Bristol

18 + 32 = 50 miles

Return home journey = 50 miles

Wyn travels 100 miles a day

5 days a week = 5 x 100 =500

Therefore, Wyn travels 500 miles each week

One day, Wyn had to use a different route through Gloucester to get to and from work.

Alternative Route

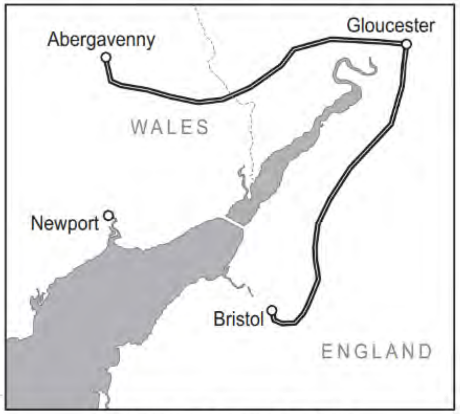


Diagram not drawn to scale

Use the chart to work out how many **extra** miles Wyn travelled that day.
You must show all your working.

Normally 100 miles a day

New route is from Abergavenny to Gloucester and then Gloucester to Bristol

45 + 36 = 81miles

Return home = 81 miles

Total distance travelled 162 miles

So, Wyn travelled 62 extra miles

Mathematics

Foundation

Unit 13

Simplifying in Algebra

Key Words:

Term: This is any part of an expression or equation that involves a letter.

e.g. $4m$, $-2r$, and p are all terms

Expression: This is a collection of terms, sometimes including numbers as well, it does not have an equals sign.

e.g. $4m + 2r$ and $8z - 5p + 6q^2 - 7$ are all expressions

Equation: This is like an expression but it contains an equals sign.

e.g. $4m + 2r = 7$ and $8z + 6q^2 - 7 = a$ are all equations

You can add or subtract LIKE TERMS, but you **cannot** add or subtract DIFFERENT TERMS.

A LIKE TERM is a term that contains the exact same letter (or letters) as another term.

For example:

$m + m = 2m$ (These are LIKE TERMS as they both have the term m)

$3p + 2p = 5p$ (These are LIKE TERMS as they both have the term p)

$16t^2 - 4t^2 = 12t^2$ (These are LIKE TERMS as they both have the term t^2, note t and t^2 are not like terms)

$3p + 2r = 3p + 2r$ (These are NOT LIKE TERMS as they both have different terms, one term is p the other term is r, so we cannot simplify them)

$3x + 2y = 3x + 2y$ (These are NOT LIKE TERMS as they both have different terms, one term is x the other term is y, so we cannot simplify them)

You will see how to do these in the examples on the next page.

Be careful, you can multiply LIKE TERMS **AND** DIFFERENT TERMS.

For example:

$m \times m = m^2$

$3p \times 2 = 6p$

$x \times y = xy$

$3p \times 2r = 6pr$

You will see how to do these in the examples over the next few pages.

Mathematics

Foundation

Unit 13

Simplifying Expressions

Note: To simplify an expression when adding or subtracting, draw boxes around all the LIKE TERMS and deal with each set of like terms on their own.
To simplify an expression when multiplying, multiply the numbers together first, then the letters.

Adding and Subtracting

Example 1: Simplify $4m + 2p - m + 6p$

$$\boxed{4m}\boxed{+\ 2p}\boxed{-\ m}\boxed{+\ 6p}$$

First draw boxes around the like terms, making sure to include the sign in front

We have:

$$4m - m = 3m$$

$$2p + 6p = 8p$$

So: $4m + 2p - m + 6p = 3m + 8p$

Note: If you cannot see a sign in front of a term then just assume it is a **plus**

Example 2: Simplify $4t^2 - 5t - 2t - 3t^2$

We have:

$$\boxed{4t^2}\boxed{-\ 5t}\boxed{-\ 2t}\boxed{-\ 3t^2}$$

Be careful with the minus signs. **Remember:** t and t^2 are different.

So: $4t^2 - 5t - 2t - 3t^2 = t^2 - 7t$

$$4t^2 - 3t^2 = t^2$$

$$-5t - 2t = -7t$$

Multiplying

Example 1: Simplify $5b \times 2c \times 3a$

Step 1: Multiply the numbers together first

$$5 \times 2 \times 3 = 30$$

Step 2: Multiply the letters

$$b \times c \times a = abc$$

Step 3: Put them together

$$5b \times 2c \times 3a = 30abc$$

Example 2: Simplify $4r \times -3p \times 3r \times q$

Step 1: Multiply the numbers together first, be careful with the negatives

$$4 \times -3 \times 3 \times 1 = -36$$

Note: There is no number in front of the q, which means it is a 1

Step 2: Multiply the letters

$$r \times p \times r \times q = pqrr = pqr^2$$

Remember: If you multiply something by itself it means you are squaring it

Step 3: Put them together

$$4r \times -3p \times 3r \times q = -36pqr^2$$

Mathematics

Foundation

Unit 13

Forming Expressions

We can form expressions for a range of problems using letters to stand for unknown values.

Example 1:

Sam's brother Tom is 3 years older than Sam. Their dad, Will, is four times as old as Sam. Form and simplify an expression for the sum of their ages.

Let us represent Sam's age by the letter x.

Sam is x years old

Tom is 3 years older than Sam

3 years older

Tom is $x + 3$ years old

Will is four times as old as Sam

4 times as old

Will is $4x$ years old

The sum of their ages is represented by

Sam's age + Tom's age + Will's age

$x \quad + \quad x + 3 \quad + \quad 4x$

Simplifying gives:

$x + x + 4x \longrightarrow 6x + 3$

Example 2:

The width of a rectangle is $2x$ cm, the length of the rectangle is 5cm less than the width. Form and simplify an expression for the perimeter of the rectangle.

The perimeter of a rectangle is found by adding all the lengths of the sides together.

Width is $2x$ cm

Length is $2x - 5$ cm

So, an expression for the perimeter is given by:

$2x + 2x + 2x - 5 + 2x - 5 \longleftarrow$ 2 widths and 2 lengths

Simplifying gives:

$2x + 2x + 2x + 2x \longrightarrow 8x - 10 \longleftarrow (-5) + (-5)$

Mathematics

Foundation

Unit 13

Expanding Single Brackets

When we expand brackets, we multiply the number/term outside the bracket by each number/term inside the bracket.

$3 \times 5a = 15a$

$3(5a - 2)$

$3 \times -2 = -6$

$3(5a - 2) = 15a - 6$

Example 1: $-3(2x + 6)$

Remember, the -3 is multiplied by everything inside the bracket.

$-3 \times 2x = -6x$

$-3(2x + 6)$

$-3 \times 6 = -18$

$-3(2x + 6) = -6x - 18$

Example 2: $-10(2c - 4)$

Remember, the -10 is multiplied by everything inside the bracket.

$-10 \times 2c = -20c$

$-10(2c - 4)$

$-10 \times -4 = 40$

$-10(2c - 4) = -20c + 40$

Example 3: $6a(2a + 6)$

Remember, the $6a$ is multiplied by everything inside the bracket.

$6a \times 2a = 12a^2$

$6a(2a + 6)$

$6a \times 6 = 36a$

$6a(2a + 6) = 12a^2 + 36a$

Example 4: $-5y(4 - 2y)$

Remember, the $-5y$ is multiplied by everything inside the bracket.

$-5y \times 4 = -20y$

$-5y(4 - 2y)$

$-5y \times -2y = 10y^2$

$-5y(4 - 2y) = -20y + 10y^2$

Mathematics

Foundation

Unit 13

Expanding Pairs of Single Brackets

When we expand pairs of single brackets, we separate the question into two parts, work each part out separately, then combine and simplify the answers.

$$3(5a-2)+2(2a+4)$$

$3(5a-2)=15a-6$ $2(2a+4)=4a+8$

$$15a-6+4a+8=19a+2$$

Example 1: $6(x+4)+2(x-7)$

Separate into two parts:

$$6(x+4)+2(x-7)$$

$6(x+4)=6x+24$ $2(x-7)=2x-14$

Combine and simplify:

$6x+24+2x-14=8x+10$

Example 2: $5(x-2)-3(x+1)$

Separate into two parts:

$$5(x-2)-3(x+1)$$

$5(x-2)=5x-10$ $-3(x+1)=-3x-3$

Combine and simplify:

$5x-10-3x-3=2x-13$

Example 3: $5(x-1)-2(x-3)$

Be careful with the minus signs

Separate into two parts:

$$5(x-1)-2(x-3)$$

$5(x-1)=5x-5$ $-2(x-3)=-2x+6$

Combine and simplify:

$5x-5-2x+6=3x+1$

Mathematics

Foundation

Unit 14

Substitution in Algebra

Substitution is where you are told the value of a letter and you substitute this into an expression or equation.

e.g. Find the value of $5x$ when $x = 7$, means 5 x x = 5 x 7 = 35.

- Always apply BIDMAS/BODMAS
- Use brackets for powers
- For fractions, work out the top and bottom separately.

Example 1: Evaluate (find the **value** of) the expressions, given that:

$a = 2,\ b = 3,\ c = -5, d = -1$

a) $5a = 5 \times 2$
$= \mathbf{10}$

b) $3b - 2c = 3 \times 3 - 2 \times (-5)$
$= 9 + 10$
$= \mathbf{19}$

c) $4b^2 + d = 4 \times 3^2 + (-1)$
$= 4 \times 9 - 1$
$= 36 - 1$
$= \mathbf{35}$

d) $3a^3 = 3 \times (2)^3$
$= 3 \times 8$
$= \mathbf{24}$

e) $\frac{5cd}{a+b} = \frac{5 \times (-5) \times (-1)}{2+3}$
$= \frac{25}{5}$
$= \mathbf{5}$

f) $c^2 + abd = (-5)^2 + 2 \times 3 \times (-1)$
$= 25 - 6$
$= \mathbf{19}$

Example 2: Evaluate (find the **value** of) the expressions, given that: (*calculator questions*)

$a = 1.2,\ b = \frac{1}{9},\ c = -3.65$

a) $4b - 6c + a^2 = 4 \times \frac{1}{9} - 6 \times (-3.65) + (1.2)^2$
$= \frac{4}{9} + 21.9 + 1.44$
$= \mathbf{23.78\dot{4}}$

b) $\sqrt{\frac{a+4c}{3b+c}} = \sqrt{\frac{1.2 + 4 \times (-3.65)}{3 \times \frac{1}{9} + (-3.65)}}$

$= \sqrt{\frac{-13.4}{-3.31\dot{6}}}$

$= \sqrt{4.0402010051}$

$= 2.0100251255$

Learn how to do these in one step using your scientific calculator.

Example 3: Use the formula $P = 5A - 6B$ to find the value of:

a) P when $A = 7$ and $B = -4$.

$P = 5A - 6B$

$P = 5 \times 7 - 6 \times (-4)$

$P = 35 + 24$

$P = 59$

b) A when $B = 3$ and $P = 37$

$P = 5A - 6B$

$37 = 5A - 6 \times 3$

$37 = 5A - 18$

$37 + 18 = 5A$

$55 = 5A$

$\frac{55}{5} = A \qquad A = 11$

Mathematics

Foundation

Unit 14

Example 4: A security firm uses the following formula to give the approximate number of staff it will need for certain events:

$$N = 0.035A + \frac{d^2}{300}$$

N is the number of staff needed.

A is the estimated number of people attending the event.

d is a measure related to the area that will need to be patrolled.

How many staff will be needed at an event where the estimated attendance is 550 and d is given as 50? Give your answer correct to the nearest whole number.
(Remember $0.035A$ means $0.035 \times A$)

$$N = 0.035A + \frac{d^2}{300}$$

$$N = 0.035 \times 550 + \frac{50^2}{300}$$

$$N = 27.5833 \ldots$$

So, $N = 28$ staff (to nearest whole number).

Example 5: Helen makes greeting cards which she sells at a weekly market. Her weekly profit (P), in pounds, is given by the formula:

$$P = 2.99S - 0.7M$$

Where S is the number of cards she sells and M is the number of cards she made.

One week she sold 60 cards but made a loss of £30.60.
How many cards had she made?
(Remember $2.99S$ means $2.99 \times S$ and $-0.7M$ means $-0.7 \times M$)

$S = 60$ and $P = -30.60$

$$P = 2.99S - 0.7M$$

$$-30.60 = 2.99 \times 60 - 0.7M$$

$$-30.60 = 179.4 - 0.7M$$

$$-210 = -0.7M$$

$$-\frac{210}{-0.7} = M$$

So, $M = 300$

Mathematics

Foundation

Unit 14

Function Machines / Number Machines

Example 1:

A number machine is shown below.

INPUT → $\div\ 2$ → $-\ 8$ → OUTPUT

a) Calculate the OUTPUT when the INPUT is 10.

Start with 10

Divide by 2 to give 5

Subtract 8 to give -3

The OUTPUT is -3

b) Calculate the INPUT when the OUTPUT is 7.

To find the INPUT from the OUTPUT use the inverse operations

(start at the end, go backwards through the number machine, do the opposite operation to the one given)

Start with 7

Add 8 (the opposite of -8) to give 15

Multiply by 2 (the opposite of $\div 2$) to give 30

The INPUT is 30

Example 2:

A number machine is shown below.

INPUT → $+\ 2$ → $\times\ 4$ → OUTPUT

a) Calculate the OUTPUT when the INPUT is 3.

Start with 3

Add 2 to give 5

Multiply by 4 to give 20

The OUTPUT is 20

b) Calculate the INPUT when the OUTPUT is 16.

To find the INPUT from the OUTPUT use the inverse operations

(start at the end, go backwards through the number machine, do the opposite operation to the one given)

Start with 16

Divide by 4 (the opposite of $\times 4$) to give 4

Subtract 2 (the opposite of $+2$) to give 2

The INPUT is 2

Mathematics

Foundation

Unit 15

Sequences

Sequence: A list which is in a particular order following a pattern.

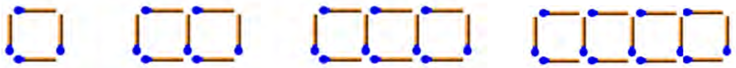
Term: Each particular part of a sequence.

Term to term rule: This is the rule for finding the next pattern in a shape, or the next number in a sequence.

Finding the Term to Term Rule

Example 1: Some matchsticks have been laid to form patterns.
The sequence can be found by counting the matchsticks in each pattern.
What is the term to term rule for the pattern?

Pattern



3 matchsticks have been added to make the next pattern.

Term to term rule **Add 3 / + 3**

Example 3: Describe the term to term rule for continuing the sequence

3, 9, 27, 81, …

The numbers in the sequence are getting bigger which implies either adding a number or multiplying by a number.

3, 9, 27, 81

+6 +18 +54

Does **not** go up by the same number each time, so look at multiplying

3, 9, 27, 81

X3 X3 X3

Multiplies by the same number each time

The rule is: multiply by 3 / x 3 / multiply the previous term by 3

Example 2: Write the next two numbers in the sequence and describe in words the rule for continuing the sequence

35, 30, 25, 20, …, …

The numbers in the sequence are getting smaller which implies either subtracting a number or dividing by a number.

35, 30, 25, 20

-5 -5 -5

Goes down by the same number each time

So, the next two numbers in the sequence are 15 and 10 (20 – 5, 15 – 5)

The rule is: subtract 5 / – 5 / subtract 5 from the previous term

Mathematics

Foundation

Unit 15

Nth Term

The nth term of a sequence is the position-to-term rule using n to represent the position number, it gives us the rule to find each term in a sequence. You may be asked to generate a sequence from an nth term or asked to find a specified term within that sequence.

Example 1 – Generating a sequence from an nth term:

The nth term for a sequence is $3n - 2$.

What are the first 3 terms of the sequence?

The "first three terms" means we are looking for the term when n is 1 (the first term), when n is 2 (the second term), and when n is 3 (the third term).

We substitute n = 1, n = 2, and n = 3 into the nth term.

So, for n = 1 $\quad 3n - 2 = 3 \times 1 - 2 = 1$

For n = 2 $\quad 3n - 2 = 3 \times 2 - 2 = 4$

For n = 3 $\quad 3n - 2 = 3 \times 3 - 2 = 7$

The first three terms of the sequence are: 1, 4, 7

Example 2 – Finding a specified term in a sequence:

The nth term for a sequence is $2n + 7$.

Find the 20th term and the 100th term.

The "20th term" means we are looking for the term when n is 20, the 100th term means we are looking for the term when n is 100.

We substitute n = 20, and n = 100 into the nth term.

For n = 20 $\quad 2n + 7 = 2 \times 20 + 7 = 47$

For n = 100 $\quad 2n + 7 = 2 \times 100 + 7 = 207$

The 20th term of the sequence is 47.

The 100th term of the sequence is 207.

Mathematics

Foundation

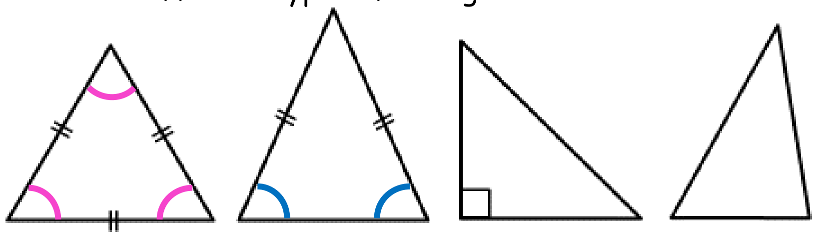
Unit 16

2-D Shapes

Different Types of 2-D Shapes and their Properties

Triangle – 3-sided shape

There are different types of triangles:

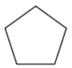


Equilateral Triangle	Isosceles Triangle	Right-angle Triangle	Scalene Triangle
3 equal lengths 3 equal angles	2 equal lengths 2 equal angles	1 right angle	No equal lengths No equal angles

Other Regular Polygons – 2-D shapes with straight sides

(Regular polygons have equal sides, an isosceles triangle, and a square are types of regular polygons)

Pentagon - 5-sided shape

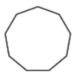


Hexagon – 6-sided shape

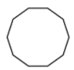
Heptagon – 7-sided shape

Octagon – 8-sided shape

Nonagon – 9-sided shape

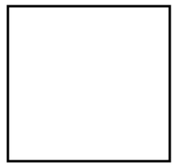


Decagon – 10-sided shape



Quadrilateral – 4-sided shape

There are different types of quadrilaterals:



Square

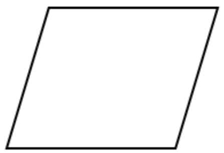
2 sets of parallel sides
All sides equal
All angles equal
4 lines of symmetry

Rectangle

2 sets of parallel sides
2 sets of equal sides
All angles equal
2 lines of symmetry

Trapezium

1 set of parallel sides
No equal sides
2 sets of equal angles
1 line of symmetry

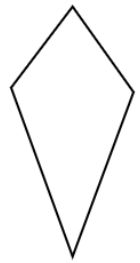


Rhombus (like a pushed over square)

2 sets of parallel sides
All sides equal
2 sets of equal angles
2 lines of symmetry

Parallelogram (like a pushed over rectangle)

2 sets of parallel sides
2 sets of equal sides
2 sets of equal angles
No lines of symmetry



Kite

No parallel sides
2 sets of equal sides
1 set of equal angles
1 line of symmetry

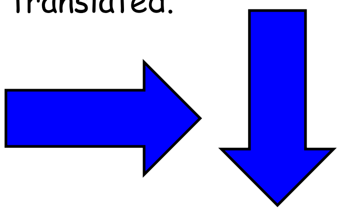
Mathematics

Foundation

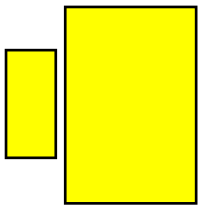
Unit 16

Congruent Shapes

Congruent shapes have the same shape and size, but could be rotated, reflected, or translated.



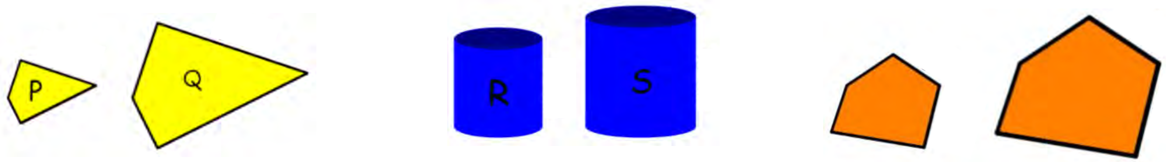
The blue arrows are congruent, they are the same shape and size; one has been rotated.



The yellow rectangles are not congruent, they are the same shape but not the same size.

Similar Shapes

Shapes are classed as similar if they are the same shape and one of them is an **enlargement** of the other.

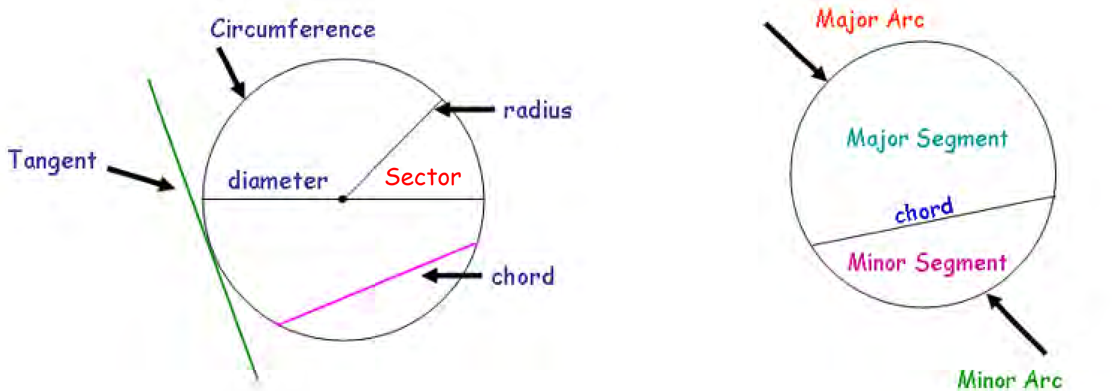


Technically, to get from one object to the other you must multiply (or divide) **every single length** by the same number

This number is called the **Scale Factor**.

Parts of a Circle

You need to know the different parts of a circle.



Mathematics

Foundation

Unit 17

Coordinates

Coordinates are given in the form (x, y), they are used to give positions on a graph.

A graph has two axes, the x-axis and the y-axis.

The point $(0, 0)$ is called the origin.

Coordinates have 2 numbers separated by a comma in a pair of brackets. E.g. $(4, -7)$

The first number is the x-coordinate and the second number is the y-coordinate.

The first number, the x-coordinate, tells you how many to go across (left if the number is negative, right if it is positive).

$- \longleftrightarrow +$

The second number, the y-coordinate, tells you how many to go up or down (down if the number is negative, up if it is positive).There are two axes on a graph (the y-axis and the x-axis).

$+$

$\updownarrow$

$-$

There are a few different ways of remembering which direction to go first (does the x-coordinate come first or the y-coordinate?).

- One way to remember which axis is which, is "x is a **cross**, so the x axis is across"
- Coordinates are written alphabetically, so the x-coordinate comes before the y-coordinate
- Another way to remember is you go along the corridor (along the x-axis) before you go up the stairs (up the y-axis).

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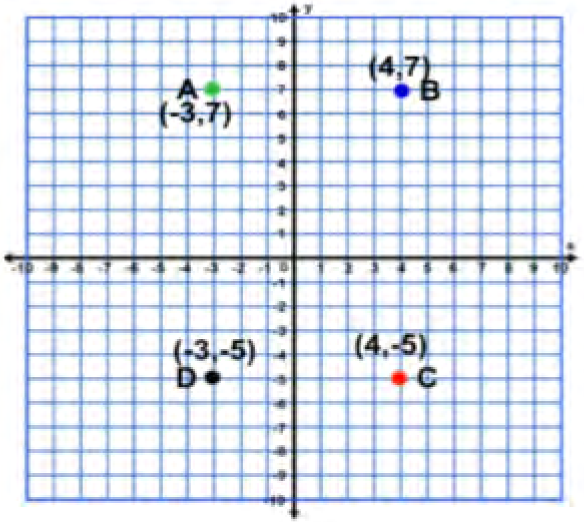
Unit 17

Plotting Coordinates

Example: Plot the points $A(-3,7)$, $B(4,7)$, $C(4,-5)$, and $D(-3,-5)$.

Point A has coordinates $(-3,7)$, to plot A go across to -3 (left) and then up to 7.

Point B has coordinates $(4,7)$, to plot B go across to 4 (right) and then up to 7.

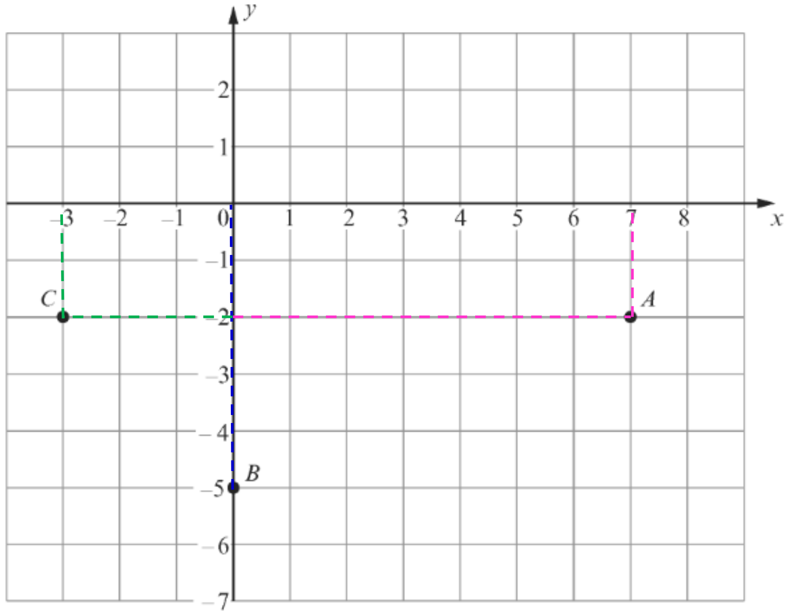


Point D has coordinates $(-3,-5)$, to plot D go across to -3 (left) and then down to -5.

Point C has coordinates $(4,-5)$, to plot C go across to 4 (right) and then down to -5.

Reading Coordinates

Example: Write down the coordinates of points A, B, and C.



Remember, read the x-coordinate first, then the y-coordinate. Write the coordinates in a bracket separated by a comma.

To get to A you go across to 7 and down to -2.
The coordinates of A are $(7,-2)$.
To get to B you stay at 0, and go down to -5.
The coordinates of B are $(0,-5)$.
To get to C you go across to -3 and down to -2.
The coordinates of C are $(-3,-2)$.

Mathematics

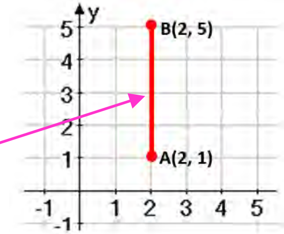
Foundation

Unit 17

Coordinates of the Mid-Point of 2 Sets of Coordinates

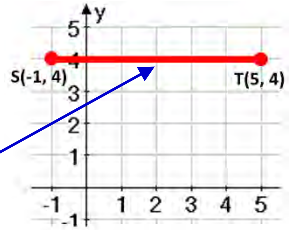
This just means finding the coordinates of the middle of a line, or the middle of two points.

Look at the example to the right, the line goes up 4 squares from A. The x-coordinate does not change. So half-way along the line would be up 2 squares from A. Mid-point$(2, 3)$.



Look at the example to the right, the line goes across 6 squares from S. The y-coordinate does not change. So half-way along the line would be across 3 squares from S.

Mid-point$(2, 4)$.

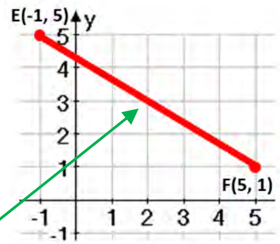


What if we had two points that were not plotted?

We would plot the points first, and then join them up with a line.

What if we had a diagonal line? Look at the example to the Right. We look at each direction in turn.
The line goes across 6 squares from E, so half-way would be across 3 squares from E. (This would take you to the x-coordinate 2). The line goes down 4 squares from E, so half-way would be down 2 squares from E. (This would take you to the y-coordinate 3).

Mid-point$(2, 3)$.



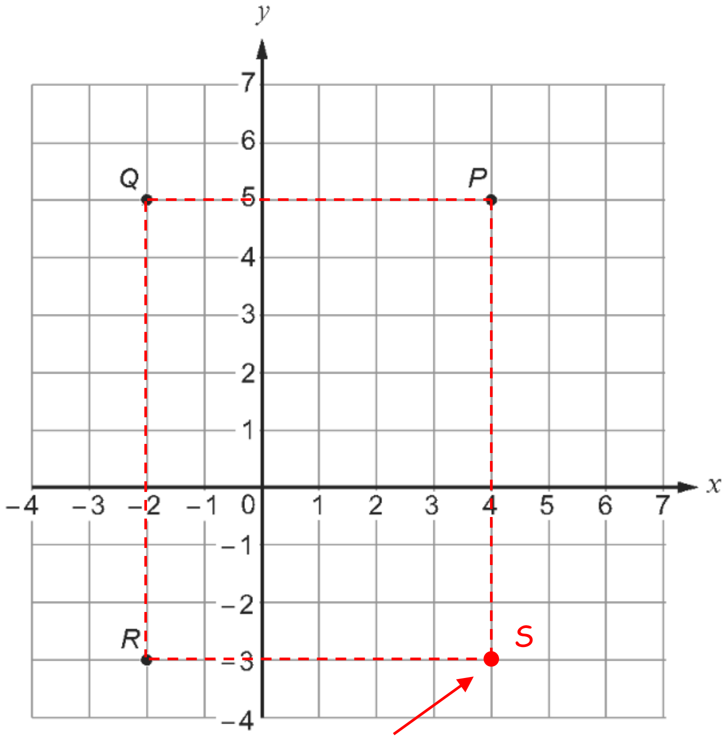
Half of how much the line goes across

Half of how much the line goes down

Finding a Coordinate of a Quadrilateral

P, Q, R, and S are the vertices of a rectangle. Plot the 4th vertex of the rectangle on the grid below and label it as the point S.

(Vertices mean corners, vertex means corner. So, P, Q, R, and S are corners of a rectangle. Plot the 4th corner.)



This is the only position to plot S so that the points make a rectangle.

Mathematics

Foundation

Unit 18

Construction

Construction is the act of drawing shapes, angles or lines accurately using a compass, protractor, and a ruler.

Drawing and Measuring Lines

Example 1: Measure the length of the line AB.

Write your answer in centimetres.

The line AB is 9.5cm long

Make sure your line starts at the zero on the ruler

Line your ruler up with the line

Read the measurement on the ruler at the end of the line 9.5cm

Make sure you are reading the cm scale

Place your ruler on the paper

Example 2: Draw a line 55mm long

Option one: Using the mm scale on a ruler

Make sure your line starts at the zero on the ruler

55mm

Draw your line to the measurement stated in the question, 55mm

Option 2: Using the cm scale on a ruler

To change mm to cm, divide by 10. So 55mm = 5.5cm

Make sure your line starts at the zero on the ruler

5.5cm

Draw your line to the measurement stated in the question, 5.5cm

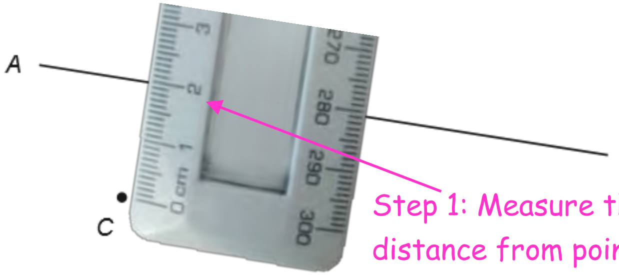
Mathematics

Foundation

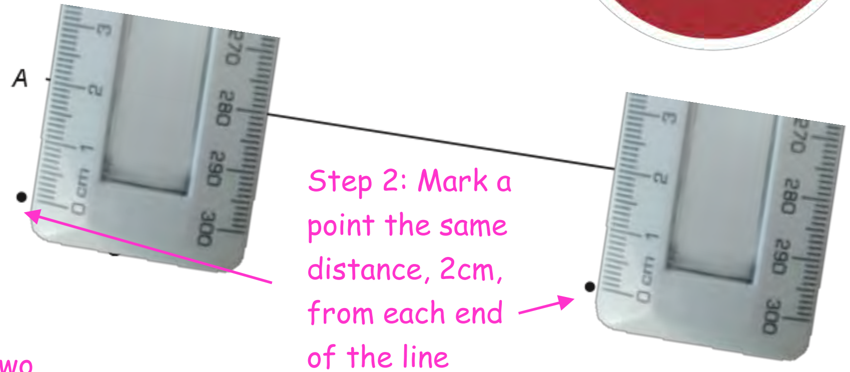
Unit 18

Drawing Parallel and Perpendicular Lines

Example 1: A line AB is shown below.
Draw a line parallel to AB that passes through point C.
A line parallel to AB will go in the same direction as AB, and always be the same distance away from AB.



Step 1: Measure the distance from point C to the line AB, 2cm

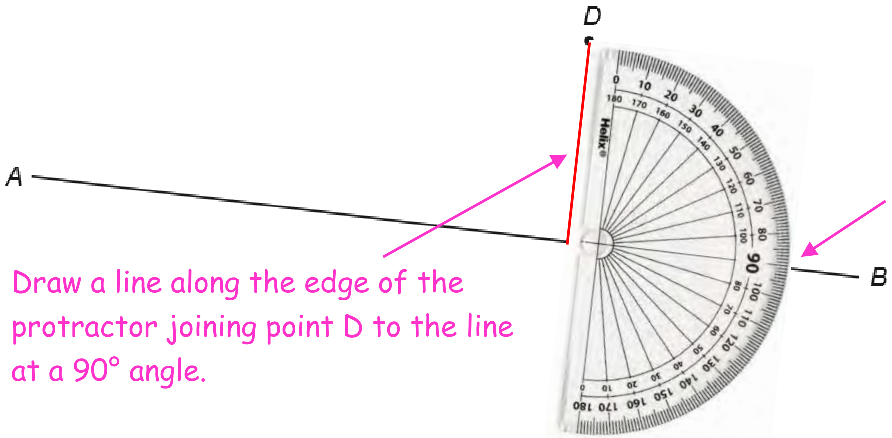


Step 2: Mark a point the same distance, 2cm, from each end of the line

Step 3: Join these two points and C with a straight line.

This line is parallel to AB

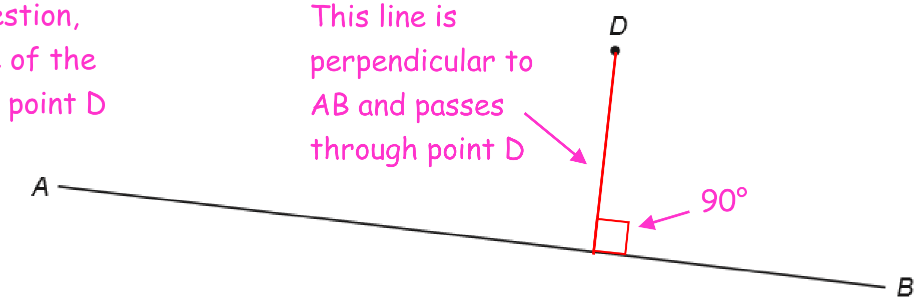
Example 2: A line AB is shown below. Draw a line perpendicular to AB that passes through point D.
A line perpendicular to AB will be at 90° to the line AB.



Line the 90° on the protractor up with the line in the question, with the edge of the protractor on point D

Draw a line along the edge of the protractor joining point D to the line at a 90° angle.

This line is perpendicular to AB and passes through point D



Mathematics

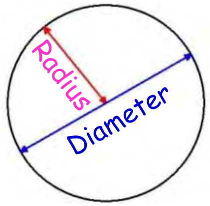
Foundation

Unit 18

Constructing Accurate Circles

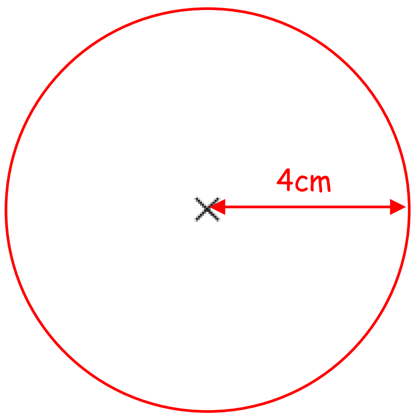
Example 1: Draw a circle of radius 4cm. Use point X as the centre of the circle.

Remember: The radius of a circle is the distance from the centre to the edge, the diameter is the distance all the way across the circle.



Open your compass the required measurement of the radius, 4cm. (If you are given the diameter, you will need to half it first).

Put the point of the compass on X, draw a circle.



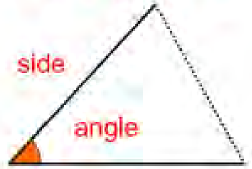
What is the diameter of your circle?

The diameter is the radius multiplied by 2. 4 x 2 = 8cm

Constructing SAS Triangles

How could we construct a triangle given the lengths of two of its sides and the angle between them?

Side Angle Side



The angle between the two sides is often called the **included angle**.

Example: Construct triangle ABC with $AB = 6$cm, $\hat{B} = 68°$, and $BC = 5$cm.

Step 1: Start by drawing side AB with a ruler.

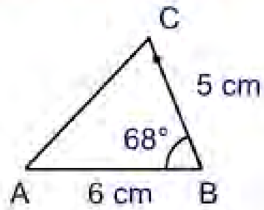
Step 2: Use a protractor to mark an angle of 68° from point B.

Step 3: Use a ruler to draw a line of 5 cm from B to C.

Step 4: Join A to C using a ruler to complete the triangle.

Check your accuracy by measuring the length of AC.

6.3 cm



Mathematics

Foundation

Unit 18

Constructing ASA Triangles

How could we construct a triangle given two angles and the length of the side between them?

Angle Side Angle

The side between the two angles is often called the **included side**.

Example: Construct triangle ABC with $AB = 5\text{cm}$, $\hat{A} = 35°$, and $\hat{B} = 115°$.

Step 1: Start by drawing side AB with a ruler.

Step 2: Use a protractor to mark an angle of 35° from point A.

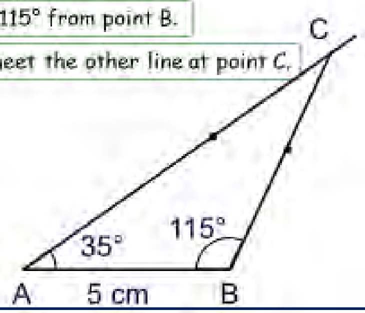
Step 3: Use a ruler to draw a long line from A.

Step 4: Use a protractor to mark an angle of 115° from point B.

Step 5: Use a ruler to draw a line from B to meet the other line at point C.

Check your accuracy by measuring the length of AC and BC.

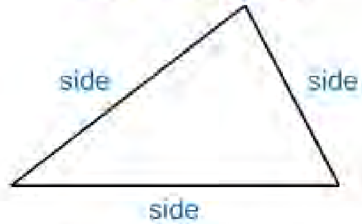
9.0 cm   5.7 cm



Constructing SSS Triangles

How could we construct a triangle given the lengths of three sides using a compass and ruler?

Side Side Side



To construct this angle you will need to use a compass.

Example: Construct triangle ABC with $AB = 4\text{cm}$, $AC = 5\text{cm}$, and $BC = 3\text{cm}$.

Step 1: Start by drawing side AB with a ruler.

Step 2: Open a pair of compasses to a length of 5cm.

Step 3: Put the compass needle at point A and draw an arc above line AB.

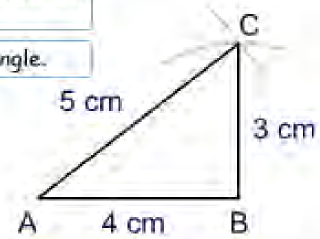
Step 4: Next, open the compasses out to a length of 3cm.

Step 5: Put the compass needle at point B and draw an arc crossing over the other one. This is point C.

Step 6: Draw lines AC and BC to complete the triangle.

Check your accuracy by measuring angle C.

54°



Mathematics

Foundation

Unit 18

Constructing quadrilaterals

Example: Construct the quadrilateral $ABCD$ with $AB = 6$cm, $\hat{A} = 100°$, $BD = 2$cm, and $\hat{B} = 120°$

Step 1: Start by drawing side AB with a length of 6cm using a ruler.

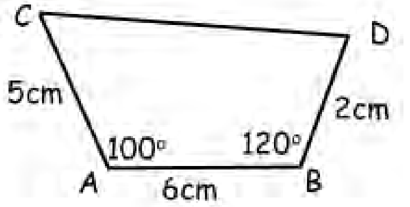
Step 2: Use a protractor to mark an angle of 100° from point A.

Step 3: Use a ruler to draw a line 5cm from A going through the marked point in step 2 and label it C.

Step 4: Use a protractor to mark an angle of 120° from point B.

Step 5: Use a ruler to draw a line 2cm from B going through the marked point in step 4 and label it D.

Step 6: Draw a line connecting C and D.



Check your accuracy by measuring length CD.

8.5cm

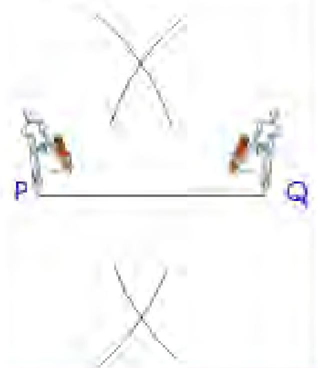
Bisecting a Line (Constructing a Perpendicular Bisector (90°))

Bisecting a line means to cut the line in half (into two equal parts at 90°)

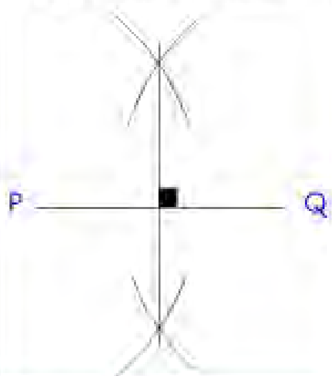
Example: Construct a perpendicular bisector to the line PQ

1. Set your compass to over half the length of the line. Place the pointy bit of the compass at P and draw an arc above and below the line:

2. Making sure you keep your compass at the exact same setting, place the pointy bit at Q and draw two more arcs.



3. With your ruler, draw a straight line through the two points where the arcs cross, and that is your perpendicular bisector!



Note: Every point on this new line is the same distance from point P as point Q

Bisecting an Angle (Constructing an Angle Bisector)

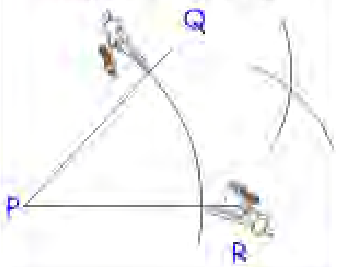
Bisecting an angle means to cut the angle in half (into two equal angles).

Example: Bisect the angle made by lines PQ and PR

1. Place the pointy bit of your compass at P and draw an arc which crosses lines PQ and PR

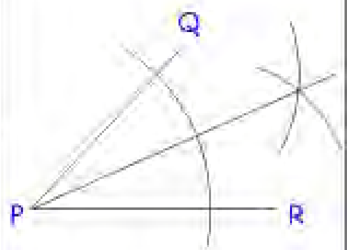
2. Place the pointy bit of the compass at both of the places where the arc hits the lines and draw two arcs

Crucial: You must not change the setting of the compass at this stage!



3. With your ruler, draw a straight line from P through the intersection of the arcs.

This is your angle bisector!



Note: Every point on this line is the same distance from line PQ as line PR

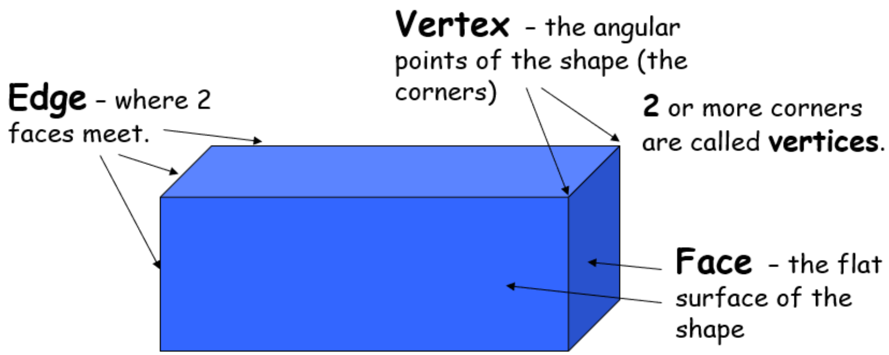
Mathematics

Foundation

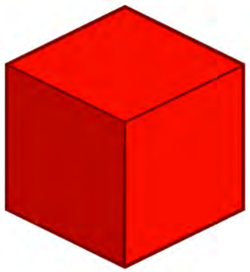
Unit 19

3-D Shapes

Properties of 3-D Shapes



Different Types of 3-D Shapes and their Properties



Cube

A cube has:

6 **equal** square faces,

12 edges,

and 8 vertices.

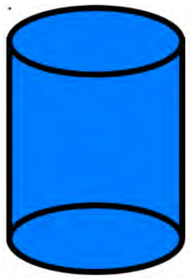
Cuboid

A cuboid has:

6 faces,

12 edges,

and 8 vertices.



Cylinder

A cylinder has:

3 faces,

2 edges,

and 0 vertices.

Cone

A cone has:

2 faces,

1 edge,

and 1 vertex.

Sphere

A sphere has:

1 face,

1 edge,

and 0 vertices.

Mathematics

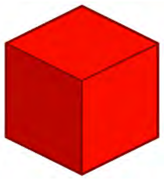
Foundation

Unit 19

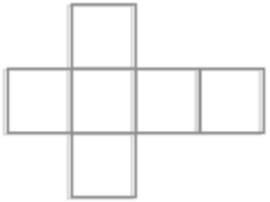
Nets of 3-D Shapes

The net of a 3-D shape is what it would look like if the shape were opened out flat.

Cube

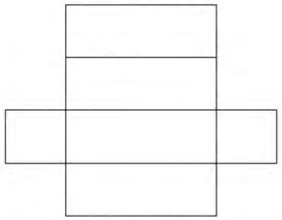


Net of a cube

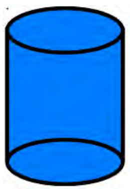


Cuboid

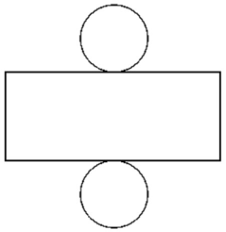
Net of a cuboid



Cylinder



Net of a cylinder



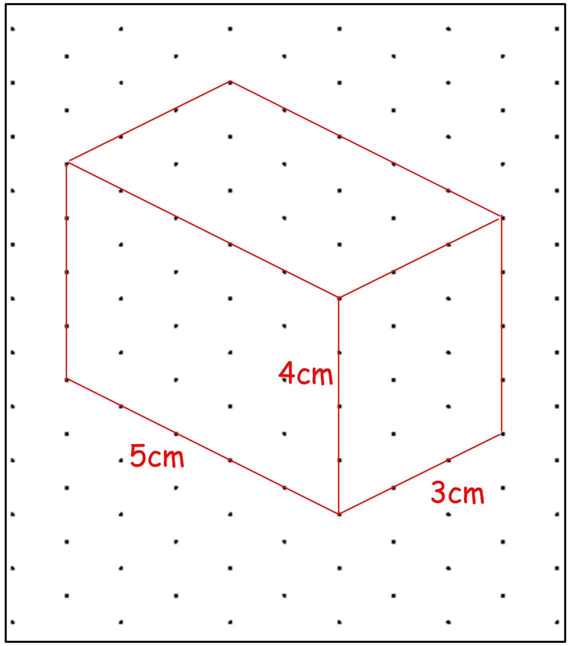
Isometric Drawing

We can draw 3-D shapes on isometric paper (dotted paper).

Example: Draw an isometric representation of a cuboid measuring 5cm by 4cm by 3cm.

The isometric paper has 1cm spaces between each dot on the diagonal. To draw the cuboid to scale we need to use lines along the diagonal dots.

Remember, when you are drawing the line you are counting the spaces between each dot, not the number of dots. So, for a line of 5cm you will count 5 spaces along.



Mathematics

Foundation

Unit 20

Data

Tally Charts

A tally chart is used to collect data; the chart is filled with marks that represent numbers. ||| = 3 ~~||||~~ = 5 ~~||||~~ || = 7

Example: 15 pupils were asked what their favourite colour. The results are shown below. Design a tally chart and put the results into the chart.

Green	Blue	Black	Green	Red
Red	Red	Black	Blue	Green
Green	Black	Red	Red	Red

For our tally chart we need a column for the colours, a column for the tally and a column for the frequency.

There are 4 greens in the above table so there are 4 tally lines

Colour	Tally	Frequency
Green	||||	4
Blue	||	2
Black	|||	3
Red	~~||||~~ |	6

Check the frequency adds to the total number of pupils, 15, if not then check the tally column again.

Pictograms

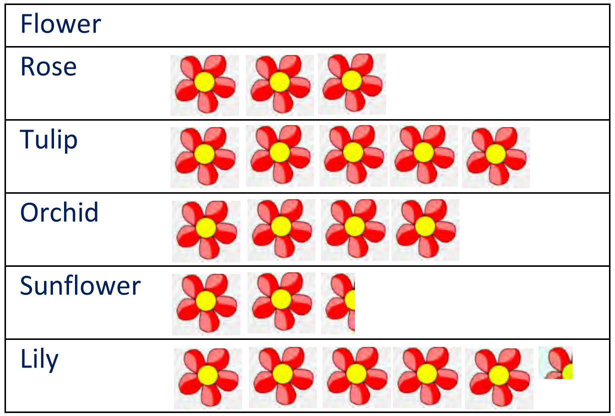
A pictogram is like a tally chart, but it uses pictures to represent the numbers rather than tally marks.

Example: Mrs Green counted the different types of flowers in her garden, the results are:

12 roses, 20 tulips, 16 orchids, 10 sunflowers, 21 lilies

Draw a pictogram to represent the number of different flowers.

Key: = 4 flowers



(4 + 4 + 4 = 12 roses)

(4 + 4 + 4 + 4 + 4 = 20 tulips)

(4 + 4 + 4 + 4 = 16 orchids)

(4 + 4 + 2 = 10 sunflowers)

(4 + 4 + 4 + 4 + 4 + 1 = 21 lilies)

Half a picture = half of 4 flowers = 2 flowers

Quarter of a picture = quarter of 4 flowers = 1 flower

Mathematics

Foundation

Unit 20

Different Types of Data

Discrete data is data that can only take on certain values, like the number of students in a class (you cannot have half a student) or shoe size (you can have size 5 or 5.5 but not 5.67).

Continuous data is data that can take on any value, like age, height, weight, temperature, or length are other examples of continuous data.

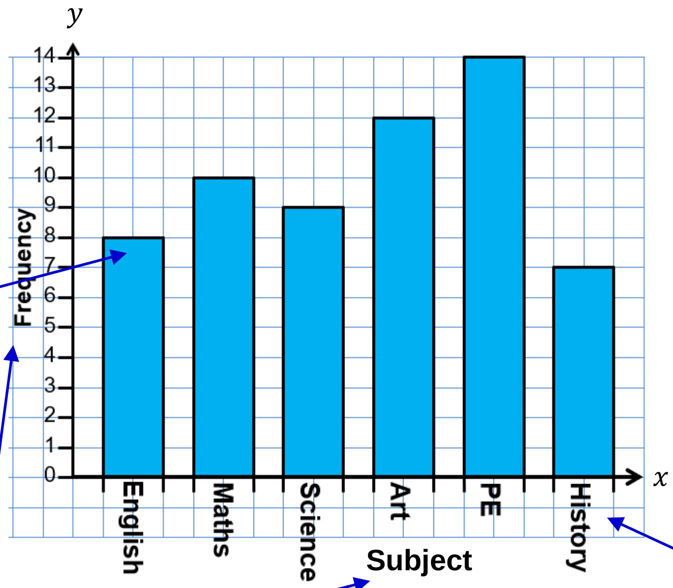
You can think of is as, Continuous data is measured, and Discrete data is counted.

Bar Chart

Example: Tom's class collected information about their favourite subjects in school. The information is shown below. Draw a bar chart to represent this information.

Favourite Subject	Frequency
English	8
Maths	10
Science	9
Art	12
PE	14
History	7

8 pupils preferred English, bar goes up to 8



Bars have equal widths
Gaps have equal widths

Bars are labelled

Axes are labelled

You could be given a bar chart (without the table) and asked to read information from it.
For example, how many pupils preferred Maths? You would look at the Maths bar and read across it to the scale on the left. 10 pupils.
For example, how many more pupils preferred PE than Science? 14 pupils preferred PE, 9 pupils preferred Science. 14 – 9 = 5. 5 more pupils preferred PE than Science.

Mathematics

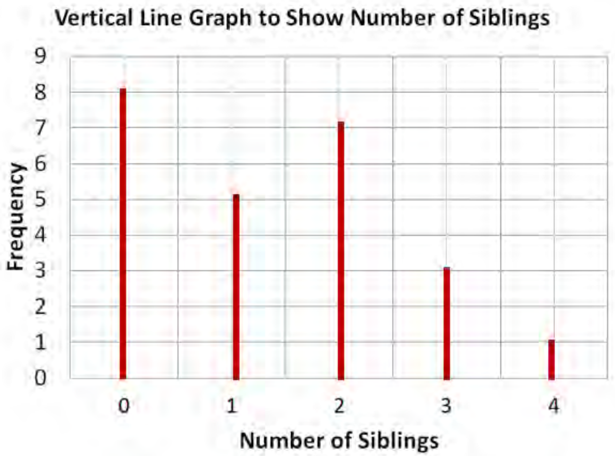
Foundation

Unit 20

Vertical Line Graph

A vertical line graph is like a bar chart, but it has thin lines instead of bars.

Example: Pupils in 11M were asked how many siblings (brothers or sisters) they had. The results are plotted in the vertical line graph below.



a) How many pupils had 0 siblings? Read across from the line for 0 siblings.

8 pupils had 0 siblings.

b) How many pupils had more than 2 siblings? We need the number of pupils with 3 siblings AND the number of pupils with 4 siblings (MORE than 2)

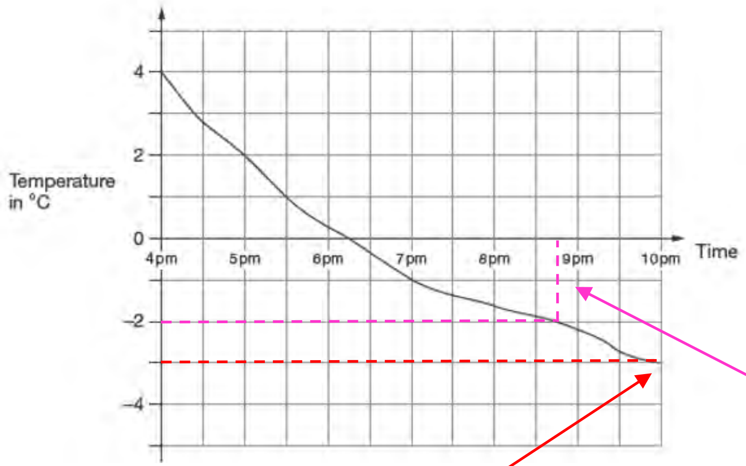
3 + 1 = 4 pupils had more than 2 siblings

c) How many pupils were asked altogether? We need the number of pupils with 0 siblings, the number of pupils with 1 sibling, the number of pupils with 2 siblings, the number of pupils with 3 siblings, and the number of pupils with 4 siblings.

8 + 5 + 7 + 3 + 1 = 24 pupils asked altogether

Temperature Charts

Example: The graph below shows the outside temperature from 4pm to 10pm on a day in winter.



a) What was the lowest temperature recorded?

$-3°C$.

b) By how much did the temperature decrease in the first hour? At 4pm the temperature was $4°C$, at 5pm the temperature was $2°C$.

The temperature decreased (fell/went down) by $2°C$

c) How far did the temperature drop between 4pm and 10pm? At 4pm the temperature was $4°C$, at 10pm the temperature was $-3°C$. (Think of a number line, the temperature has gone from +4 to -3, it has gone down 7)

The temperature dropped $7°C$

d) Estimate the time when the temperature was $-2°C$. (shown by the pink line)

Approximately 8:45 pm / 20:45 / quarter to 9

Mathematics

Foundation

Unit 20

Pie Charts

Pie charts use angles to represent proportionally the quantity of each group involved.
Pie charts can only be compared to one another when populations are given.

Example 1 - Drawing Pie Charts:

A group of 72 maths teachers were asked to choose their favourite TV show from a list, and their responses are shown in the table on the right. Construct a pie chart to illustrate this information.

TV Show	Total
Lost	12
Heroes	10
Desperate Housewives	4
Countdown	15
Teachers TV	13
The Beauty of Maths	18

Working out the Angles

- Before you can start to draw the pie chart, you need to know how big a slice each of the choices is going to take up – in other words, you need to know the angle of each segment
- To work this out, you need to remember that there are 360 degrees in a circle
- That means there are 360 degrees to share between each of the people who took part in the survey
- How many degrees does each person get? Divide 360 by the total number of people surveyed.

We have a total of 72 teachers who were surveyed. 360 ÷ 72 = 5

Each teacher is worth 5 degrees on our pie chart.

We now need to work out what angle each segment (each TV show) gets.

Note: The overall total may NOT be a factor of 360.

72 is a factor of 360 (it goes into perfectly) so we just had to multiply by 5 to get each angle.

If the total is not a factor of 360, try using this method for working out the angles:

$Lost\ \frac{12}{72} \times 360 = 60^o$ and $Heroes\ \frac{10}{72} \times 360 = 50^o$

TV Show	Total	Working Out	Angle of Segment
Lost	12	12 x 5 = 60	60°
Heroes	10	10 x 5 = 50	50°
Desperate Housewives	4	4 x 5 = 20	20°
Countdown	15	15 x 5 = 75	75°
Teachers TV	13	13 x 5 = 65	65°
The Beauty of Maths	18	18 x 5 = 90	90°

Remember: Check this column adds up to 360 before you move on.

Mathematics

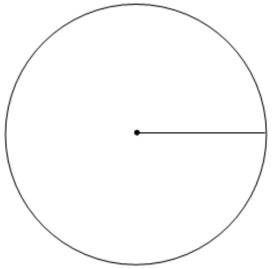
Foundation

Unit 20

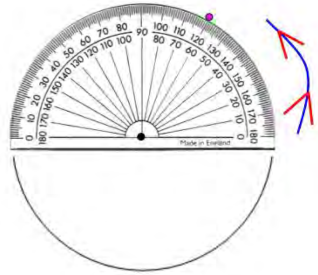
Example 1 - Drawing Pie Charts Continued:

Drawing the Pie Chart

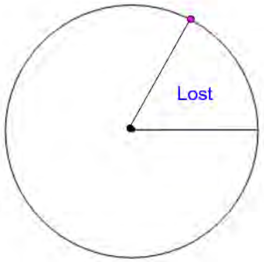
1. Draw a circle using a compass. Mark the centre with a dot and draw a straight line from the centre up to the right of your circle



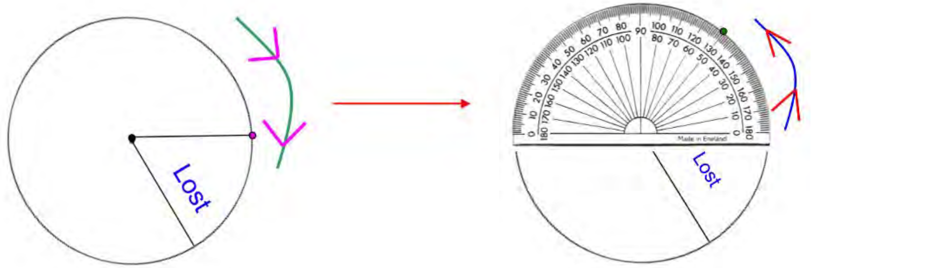
2. Carefully place your angle measurer along the line, with the **centre exactly on the centre of the circle.** Now, count around from 0 until you reach the correct number of degrees – in this case 60° – and place a dot



3. Join up your dot to the centre with a straight line and label your segment.

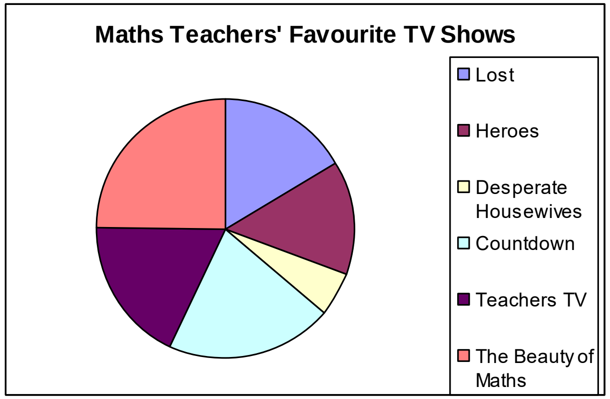


4. Turn your pie chart clockwise until your new line is horizontal (where the first line used to be). Now you can mark your next angle in the same way.



5. Keep doing this until you have drawn all your segments

6. If you want to you can colour in your segments, but you must remember to label them clearly, or add a key.



Check: You will know if you have got it right if the line to make your final segment is the very first line you drew.

Mathematics

Foundation

Unit 20

What CAN'T we tell from Pie Charts?

If we were just given the pie chart (and no original data) and were asked "how many maths teachers said that Countdown was their favourite show?", there would be no way of knowing what the answer was.

Unless we are told how many people were surveyed all together, we cannot answer that question.

When making statements based on Pie Charts, just make sure what you are saying is, 100% true.

What CAN we tell from Pie Charts?

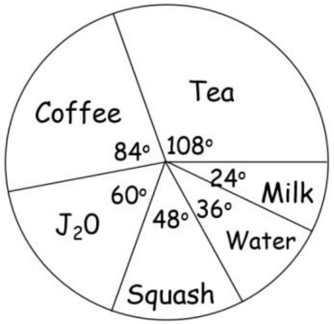
If you look back at the pie chart in the last example, you will see The Beauty of Maths was the most popular choice amongst our maths teachers, whereas Desperate Housewives was the least popular.

You could also say something like "roughly 3 times as many teachers preferred Lost to Desperate Housewives".

Example 2 - Interpreting Pie Charts:

240 Maths teachers were asked "what is your favourite drink?" and the pie chart below was drawn to show the information.

Work out how many teachers preferred coffee.



To answer this question, we must do the opposite of what we did when we were drawing the pie chart – we must use our angles to find our totals.

Let us look at the coffee segment, it takes up 84° out of 360°, and what we want to know is "how much does it take up out of our 240 people?"

$$\frac{84}{360} = \frac{?}{240} \xrightarrow{\text{Multiply both sides by 240}} \frac{84}{360} \times 240 = \textbf{56 people}$$

Note: Sometimes the angles will not be given so you would have to use a protractor to measure each section.

If a percentage was given instead of an angle, for example 30% preferred Tea, to work out how many teachers this is, you would use a similar method.

$\frac{30}{100} \times 240 = 72$ (Use 100 instead of 360 because percentages are out of 100)

Mathematics

Foundation

Unit 21

Basic Probability

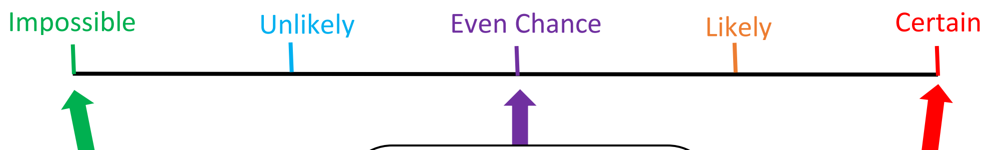
Probability is the likelihood that an event will occur.

Probabilities are always written as fractions, decimals, or percentages.

Probabilities have values between 0 and 1.

Probability scale

Probabilities can be described using words, and represented on a probability scale



What is the probability that Winter follows Summer?
Impossible

What is the probability that if you toss a coin it will land on heads?
Even Chance (the coin is equally likely to land on heads or tails)

What is the probability that Christmas will be on the 25th of December?
Certain

Probabilities can also be described using numbers

Example: A box contains the following 8 cards. They are identical except for the numbers written on them.

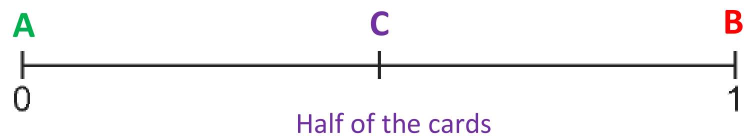
5 6 6 6 7 7 7 7

One card is chosen at random from the box. On the probability scale shown below, mark the points A, B, and C.

A is the probability that the chosen card has the number 3 on it

B is the probability that the chosen card has a number greater than 2 on it

C is the probability that the chosen card has a number less than 7 on it



There are no 3's on the cards so the probability is 0 (impossible)

Half of the cards have a number less than 7 so the probability is a half (even chance)

All the cards are greater than 2 so the probability is 1 (certain)

Mathematics

Foundation

Unit 22

Perimeter, Area, and Volume

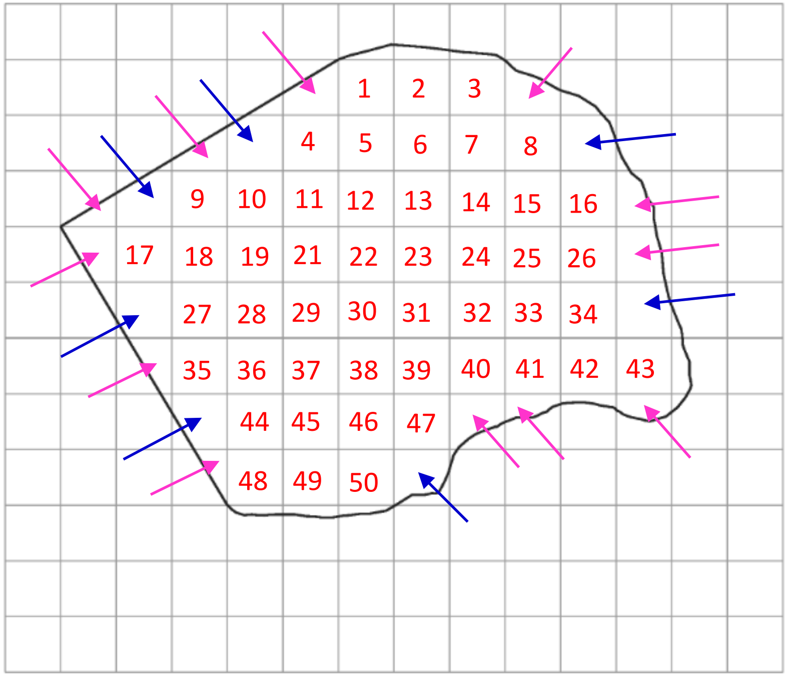
The **area** of a 2D shape is the space inside it. It is measured in units squared e.g. cm^2

The **perimeter** of a shape is the distance around the edge of the shape. Units of length are used to measure perimeter e.g. mm, cm, m

A **compound shape** is a shape made from other shapes joined together.

We can find the area of a shape by using formulas or by counting squares.

Example 1: The below shape is the outline of a field. It is drawn on a square grid where each square represents $1m^2$. Estimate the area of the field.



Each whole square represents $1m^2$.
We can estimate 2 half squares to be $1m^2$.
We can discard squares with only a small amount.
We can count squares which are largely covered as being $1m^2$.
Alternatively, we could estimate a square with a small amount and a square which is largely covered together to be $1m^2$.

There are 50 full squares. $50m^2$
There are approximately 7 squares largely covered (shown by blue arrows). $7m^2$
There are approximately 12 half squares (shown by pink arrows). $6m^2$

An estimate for the area of the field would be:

$$50 + 7 + 6 = 63m^2$$

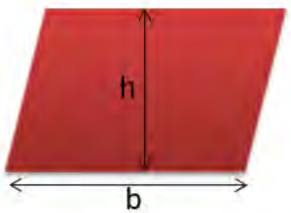
Mathematics

Foundation

Unit 22

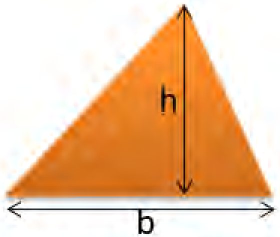
Formulas for Area:

Parallelogram



A = b × h

Triangle



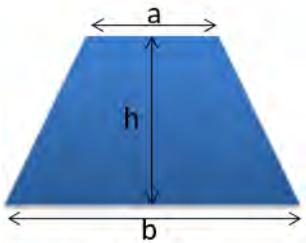
$A = \frac{b \times h}{2}$

Note: You must use the perpendicular height

Rectangle / Square

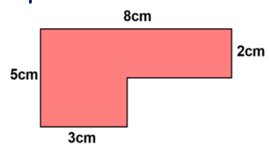
A = l × w

Trapezium



$A = \frac{(a + b) \times h}{2}$

Example 2: Find the perimeter and area of the compound shape



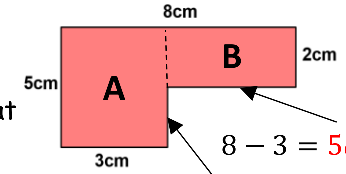
Area

Step 1: Split the shape into shapes that you can find the area of

Step 2: Find the missing lengths of sides

Step 3: Work out the area of each shape

Step 4: Work out the total area, remembering the units



$8 - 3 = 5cm$

$5 - 2 = 3cm$

$\text{Area A} = (5 \times 3)$
$= 15cm^2$

$\text{Area B} = (2 \times 5)$
$= 10cm^2$

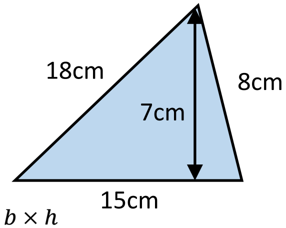
$\text{Total area} = 15 + 10$
$= 25cm^2$

Perimeter

Add up all the outside edges

$\text{Perimeter} = 3 + 5 + 8 + 2 + 5 + 3$
$= 26cm$

Example 3: Find the area of the triangle

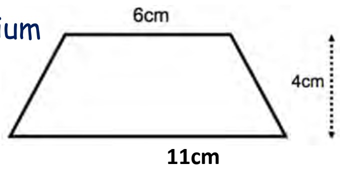


$\text{Area} = \frac{b \times h}{2}$

$= \frac{15 \times 7}{2}$

$= 52.5cm^2$

Example 4: Find the area of the trapezium



$\text{Area} = \frac{(a + b) \times h}{2}$

$= \frac{(6 + 5) \times 4}{2}$

$= 22cm^2$

Mathematics

Foundation

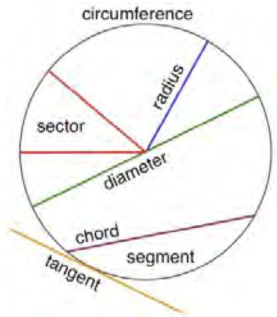
Unit 22

The circumference of a circle is the distance around the outside of the circle and is calculated using the formula:

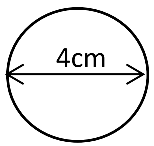
$$Circumference = \pi \times d$$

The area of a circle is calculated using the formula:

$$Area = \pi \times r^2$$



Example 1: Find the circumference of a circle with a diameter of 4cm.



C = $\pi \times 4$

= 4π

or = 12.57cm

Example 2: Find the diameter of a circle with a circumference of 20cm.

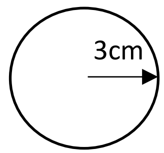
C = $\pi \times d$

20 = $\pi \times d$

$\frac{20}{\pi} = d$

$6.37cm = d$

Example 3: Find the area of a circle with radius 3cm



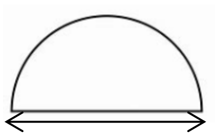
A = $\pi \times r^2$

= $\pi \times 3^2$

= 9π

or = 28.3cm^2

Example 4: Find the perimeter of the semicircle.



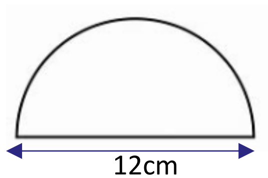
$$P = \frac{\pi \times d}{2} + d$$

$$P = \frac{\pi \times 6}{2} + 6$$

$$P = 3\pi + 6$$

P = 15.42cm

Example 5: Find the area of the semicircle.



$$A = \frac{\pi \times r^2}{2}$$

$$A = \frac{\pi \times 6^2}{2}$$

$$A = 56.55cm^2$$

Example 6: Find the radius of a circle when the area is 20cm^2.

A = $\pi \times r^2$

20 = $\pi \times r^2$

$\frac{20}{\pi} = r^2$

$\sqrt{\frac{20}{\pi}} = r$

$2.52cm = r$

Mathematics

Foundation

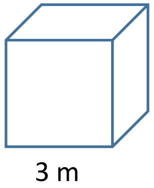
Unit 22

Volume

The **volume** of an object is the amount of space that it occupies.

It is measured in units cubed e.g. cm^3.

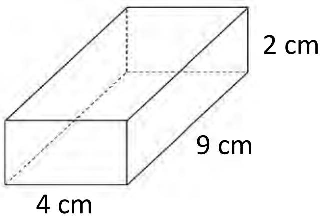
Example 1: Find the volume of the cube.



A cube has sides of equal length

$$\textbf{Volume} = length \times width \times height$$
$$= 3 \times 3 \times 3$$
$$= \mathbf{27m^3}$$

Example 2: Find the volume of the cuboid.



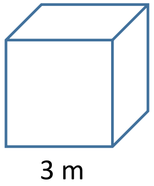
$$\textbf{Volume} = length \times width \times height$$
$$= 9 \times 4 \times 2$$
$$= \mathbf{72cm^3}$$

Surface Area

The **surface area** of an object is the area of each face added together.

It is measured in units squared e.g. cm^2.

Example 1: Find the surface area of the cube.



A cube has 6 equal faces

Surface area

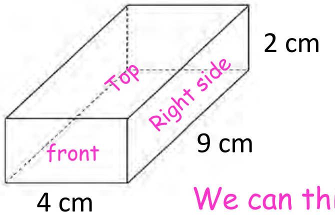
Area of one face:

$$3 \times 3 = 9m^2$$

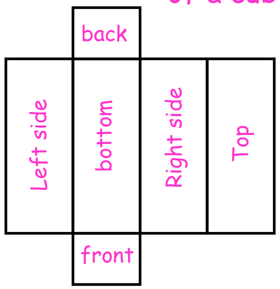
Area of six faces:

$$6 \times 9 = 54m^3$$

Example 2: Find the surface area of the cuboid.



We can think of the net of a cuboid to help us



Surface area

Area of each face:

$$Front = 4 \times 2 = 8$$
$$Back = 4 \times 2 = 8$$
$$Left\ Side = 9 \times 2 = 18$$
$$Right\ Side = 9 \times 2 = 18$$
$$Bottom = 4 \times 9 = 36$$
$$Top = 4 \times 9 = 36$$
$$Total = 8 + 8 + 18 + 18 + 36 + 36$$
$$= \mathbf{126cm^2}$$

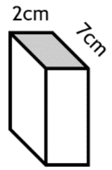
Mathematics

Foundation

Unit 22

Problem Solving

Example 1: The volume of this cuboid is 56cm^3.
Find the height of the cuboid



$$Volume\ of\ cuboid = l \times w \times h$$

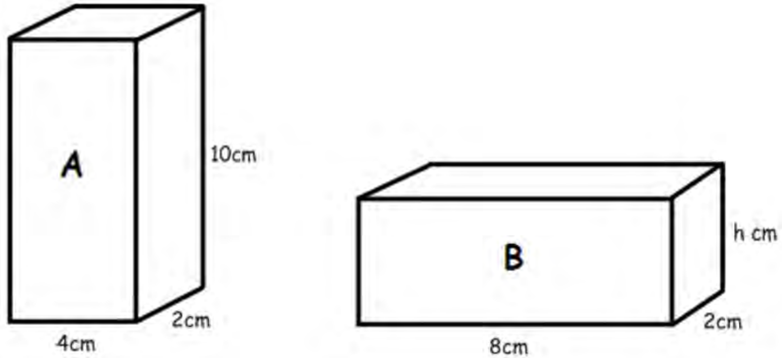
$$56 = 7 \times 2 \times h$$

$$56 = 14 \times h$$

$$\frac{56}{14} = h$$

So, $h = 4cm$

Example 2: The two cuboids A and B, each have the same volume.



Work out the height, $h\ cm$ of cuboid B.

Volume of A: $Volume = 4 \times 2 \times 10$

$$= 80cm^3$$

Volume of B: $80 = 8 \times 2 \times h$

$$80 = 16h$$

$$\frac{80}{16} = h$$

So, $h = 5cm$

Mathematics

Foundation

Unit 23

Solving Simple Equations

The aim of solving an equation is to find the value of the unknown which makes the equation balance, e.g. equation: $x - 5 = 3$, solution: $x = 8$, because $8 - 5 = 3$.

There are different methods you can use to solve equations using your knowledge of inverse operations.

An operation is a mathematical process such as adding, multiplying, or squaring, etc.

An inverse operation is the process of reversing the operation (the opposite process).

For example, when adding, the inverse operation would be subtracting, when multiplying the inverse operation would be division and so on.

Here are the main inverse operations you need to know:

$+ \rightleftarrows -$

Addition is the opposite of subtracting.
Subtracting is the opposite of adding.
They are inverse operations.

$\times \rightleftarrows \div$

Multiplication is the opposite of division. Division is the opposite of multiplication
They are inverse operations.

Mathematics

Foundation

Unit 23

Method 1: Using our knowledge of inverse operations we can rearrange the equation to get the letter (this is often x) on its own. Many teachers say this is called the "Change the side, change the sign" method.

Golden Rule: When rearranging an equation and moving a term over the equals sign to the **opposite side** it changes to the **opposite sign** (the inverse). For example, '+3' becomes '-3', or '÷4' become 'x4'.

Note: The subject term is the letter used in the equation.

Step 1: Get rid of any square root signs by squaring both sides. Clear any fractions by cross-multiplying up to every other term. Multiply out any brackets.

Step 2: Collect all subject terms on one side of the equals sign and all non-subject terms on the other. Remembering the rule "change sides, change sign" (you most often see the letters on the left-hand side and numbers on the right).

Step 3: Simplify like terms on each side of the equation.

Step 4: If you are left with a number multiplied by your subject term equals something ($Ax = B$ where A and B are numbers and x is the subject term), then to get the subject term on its own, move the number over the other side of the equals sign remembering to change its sign to the opposite sign (the inverse) which in this case is from a multiply to a divide ($Ax = B$ becomes $x = \frac{B}{A}$).

Check your answer using substitution to make sure you are right.

Example 1: $p + 7 = 32$

$$p + 7 = 32$$

Move the $+7$ over the equals sign to the opposite side and change the sign to the opposite / inverse operation

$$p = 32 - 7$$

$$p = 25$$

Example 2: $r - 12 = 36$

$$r - 12 = 36$$

Move the -12 over the equals sign to the opposite side and change the sign to the opposite / inverse operation

$$r = 36 + 12$$

$$r = 48$$

Example 3: $\frac{k}{5} = -1$

$$\frac{k}{5} = -1$$

Move the $\div 5$ over the equals sign to the opposite side and change the sign to the opposite / inverse operation

$$k = -1 \times 5$$

$$k = -5$$

Example 4: $3m = 18$

$$3m = 18$$

Remember, $3m$ means $3 \times m$

Move the $\times 3$ over the equals sign to the opposite side and change the sign to the opposite / inverse operation

$$m = \frac{18}{3}$$

$$m = 6$$

Mathematics

Foundation

Unit 23

Method 2: Balancing equations

Golden Rule: Whatever you do to one side of the equation, you must do exactly the same to the other side to keep the equation in balance

Step 1: If they are not already, get all your unknown letters on one side of the equation (NOT on the bottom of fractions and avoiding negatives).

Step 2: Begin undoing the operations that were done to your unknown letter, by thinking about the order that things were done to the letter

Step 3: Use inverse operations to do this until you are left with just your unknown letter on one side, and the answer on the other

Step 4: Check your answer using substitution to make sure your answer is right.

Example 1: $p + 7 = 32$

Undo the operations, the only operation was +7

So, subtract 7 from both sides

$$p + 7 = 32$$
$$-7 \qquad\qquad -7$$
$$p = 25$$

Check: Substitute $p = 25$ into the original equation.

$$25 + 7 = 32$$

Example 2: $r - 12 = 36$

Undo the operations, the only operation was -12

So, add 12 to both sides

$$r - 12 = 36$$
$$+12 \qquad\qquad +12$$
$$r = 48$$

Check: Substitute $r = 48$ into the original equation.

$$48 - 12 = 36$$

Example 3: $\frac{k}{5} = -1$

Undo the operations, the only operation was ÷ 5

So, multiply both sides by 5

$$\frac{k}{5} = -1$$
$$\times 5 \qquad\qquad \times 5$$
$$k = -5$$

Check: Substitute $k = -5$ into the original equation.

$$-\frac{5}{5} = -1$$

Example 4: $3m = 18$

Undo the operations, the only operation was × 3

So, divide both sides by 3

$$3m = 18$$
$$\div 3 \qquad\qquad \div 3$$
$$m = 6$$

Check: Substitute $m = 6$ into the original equation.

$$3 \times 6 = 18$$

Mathematics

Foundation

Unit 24

Averages and Dispersion

The main averages are the **mean**, **mode** and **median**.
The range is not an average but a measurement of **spread of data**.
The smaller the range the more consistent the data.

The mean

The mean uses all the values in the data.
To calculate the mean:

- Add up all of the items
- Divide by how many items there are

The mode

The mode is the most common value that appears in the data and there can be more than one.
If all the values appear the same number of times, then **there is no mode**.
Ordering the numbers can be helpful.

The median

The median is the middle value in the sorted set of data. To calculate the median:

- List the values in order from smallest to largest (ascending order)
- Cross values off from each end to identify the middle value

If there are two numbers in the middle, we add them up and divide by 2 to find the middle of those two numbers.

The range

The range is found by calculating the difference between the highest and lowest value.

Example 1: Find the mean, mode, median, and range of the following set of numbers:

10, 2, 3, 5, 15, 19, 21, 5

Mean: $\frac{2+3+5+5+10+15+19+21}{8} = \frac{80}{8} = 10$

Mode: 5

Median: ~~2~~, ~~3~~, ~~5~~, 5, 10, ~~15~~, ~~19~~, ~~21~~ $\frac{5+10}{2} = 7.5$

Range: 21 – 2 = 19

Example 2: Finding the total when given the mean of a set of numbers

The mean of a set of 6 numbers is 5. What is the total of the 5 numbers?

Remember, to find the mean $\frac{total\ value\ of\ items}{number\ of\ items} = mean$

Therefore, to find the original total of the numbers we use

$total\ value\ of\ items = mean \times number\ of\ items$

$total\ value\ of\ items = 5 \times 6 = 30$

Mathematics

Foundation

Unit 24

Example 3: Ben, Jacob, Caitlin, and Bethan were practising their football skills.

Ben scored 5 goals.

Jacob scored 7 goals.

Caitlin scored 4 goals.

The mean number of goals scored by all four players was 6.

How many goals did Bethan score?

Remember, to find the mean we use: $\frac{total\ value\ of\ items}{number\ of\ items} = mean$

Therefore, to find the total number of goals scored (total value of items) we use:

$$total\ value\ of\ items = mean\ \times\ number\ of\ items$$

$$total\ value\ of\ items = 6\ \times 4 = 24$$

If we added up each players number of goals scored, it should add to 24.

5 + 7 + 4 = 16

24 – 16 = 8

So, Bethan scored 8 goals.

Real Life Use

We can use the mean or the median to compare two distributions.

Example: 19 runners complete a marathon. 10 of them are professional athletes. 9 of them are amateur athletes.

The mean time for the professional athletes to complete the marathon was 142.4 minutes.
The mean time for the amateurs to complete the marathon was 159.6 minutes.

As the mean time for the professional athletes, 142.4 minutes, is less than the mean time for the amateur athletes, 159.6 minutes, it implies that the professional athletes are faster than the amateur athletes.

Mathematics

Foundation

Unit 24

Finding the Mean, Median and Mode from a Frequency Table

Example: A team plays 20 games; the coach records the number of goals they score in each game in a frequency table.

Number of Goals	Frequency	fx
0	5	0 x 5 = 0
1	6	1 x 6 = 6
2	4	2 x 4 = 8
3	3	3 x 3 = 9
4	2	4 x 2 = 8
	$Total\ f = 20$	$Total\ fx = 20$

Mean: To find the mean, you need to find the total value of all the data, then divide by the total frequency.

Mean goals = $\frac{Total\ fx}{Total\ f} = \frac{31}{20} = 1.55$

Median: To find the median, we need to work out what position in the data the median will be. If there are n pieces of data, the median value will be in position $\frac{n+1}{2}$.

In this case the median position is $\frac{20+1}{2} = 10.5^{th}$

The first row covers the first 5 positions so the 10.5^{th} position would be in the second row; therefore, the Median is 1 goal.

Mode: The mode is the group that contains the highest frequency.
Therefore, the mode is 1 goal.

Modal Class from a Grouped Frequency Table

Example: The frequency table shows pupils ages. Find the modal class of the pupils ages.

Ages	Frequency
8 – 10	12
11 – 13	25
14 – 16	37
17 - 19	14

The **modal class (group)** is the class with the highest frequency.

We cannot say what the single most frequent value was, but we can say which group had the most data in it, the highest frequency.

In the table to the left, the highest frequency is 37, so the modal age group is 14-16.

Mathematics

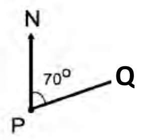
Foundation

Unit 25

Scale Drawings and Bearings

Scales are used to reduce real world dimensions to a useable size.

A **bearing** is an angle, measured from the **north** line in a **clockwise** direction. It is given as a **3-digit** number.



The bearing of Q from P is given as 070°. The easiest way of thinking about it is you are standing at P, facing north what angle would you need to turn to face Q.

You may need to draw the north line directly upwards before constructing a bearing.

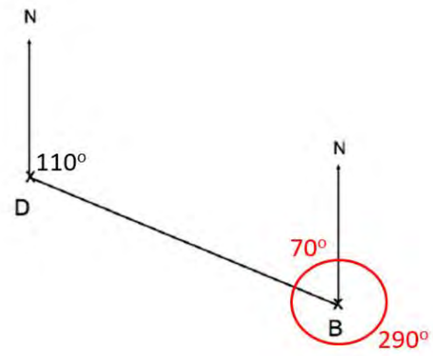
For bearing questions where diagrams are drawn to scale, you will need to use a protractor to measure or draw angles and use a ruler for measurements.

For bearing questions where diagrams are not drawn to scale, you will need to recall certain facts about angles, such as:

- Angles on a straight line add up to 180°.
- Angles around a point add up to 360°.
- Interior angles ("C" angles) add up 180°.
- Alternate angles ("Z" angles) are equal.
- Corresponding angles ("F" angles) are equal.

Example 1:

The diagram shows the position of a boat B and dock D. The scale of the diagram is 1cm to 5km.



a) Calculate the real distance between the boat and the dock.

The length from D to B measures 4.5cm

$4.5 \times 5 = 22.5km$

b) State the bearing of the boat from the dock.

110^o (from D to B)

c) Calculate the bearing of the dock from the boat.

$180^o - 110^o = 70^o$ (the angles are interior "C" angles)

$360^o - 70^o = 290^o$ (angles around a point equal 360°)

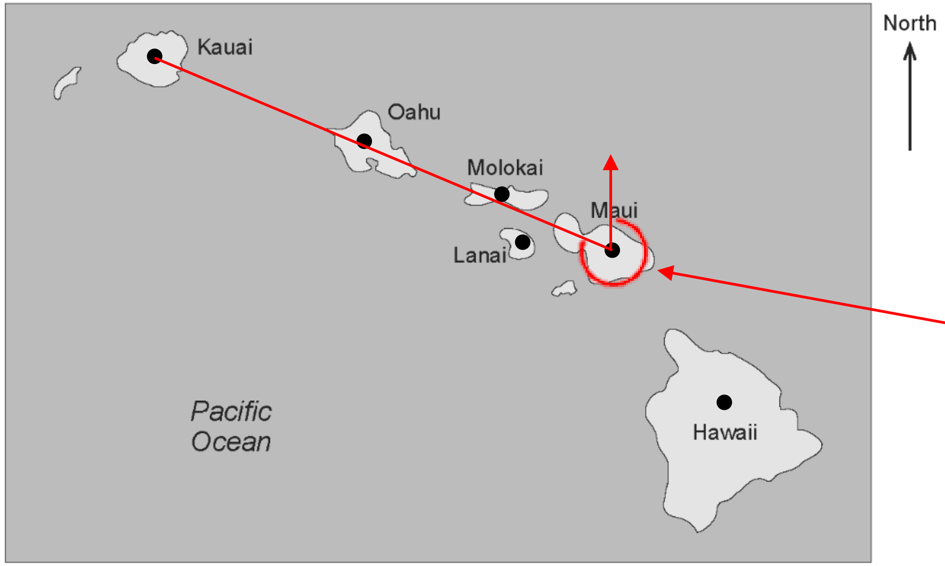
Mathematics

Foundation

Unit 25

Example 2:
The state of Hawaii in the USA consists of 8 main islands. The six largest of these islands are Hawaii, Maui, Oahu, Kauai, Molokai, and Lanai.

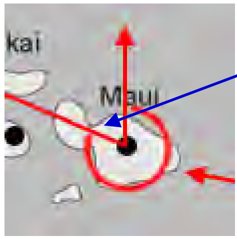
What is the bearing of the island of Kauai <u>from the island of Maui</u>?



Look for the key word in the question, this tells us where we are measuring the bearing from, and where we start.

We start at the island of Maui, there is no North line, so we draw one in. As we are measuring the bearing of the island of Kauai from the island of Maui, we need to join them up with a line.
Now we need to make sure we measure the right part, we are going <u>from</u> Maui, and the bearing we want is in the clockwise direction from the North line.

Using a protractor, we can measure the smaller angle, and subtract it from 360° to find the larger angle – the bearing.



$67°$

So, the bearing is $360 - 67 = 293°$

Mathematics

Foundation

Unit 25

Example 3: The diagram is a sketch of Swansea bay with the positions of Mumbles, Swansea and Porthcawl marked.

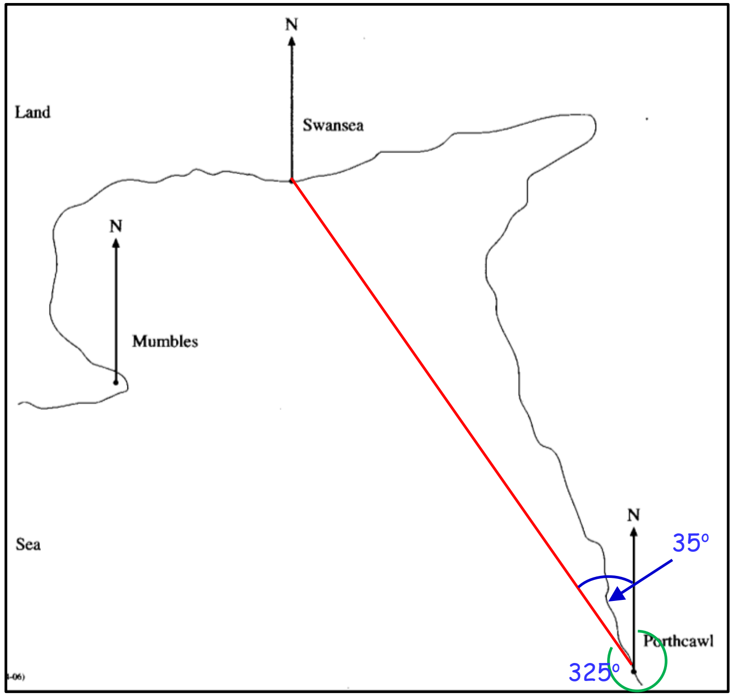
a) Find the bearing of Swansea from Porthcawl.

First, join up Swansea and Porthcawl with a straight line.

Like in example 2, measure the acute angle (smaller angle) anticlockwise from the north line and subtract from 360°. Then Subtract it from 360° to find the reflex angle (larger angle) Using a protractor this acute angle measures 35°.

$$360° - 35° = 325°$$

So, the bearing of Swansea from Porthcawl is 325°

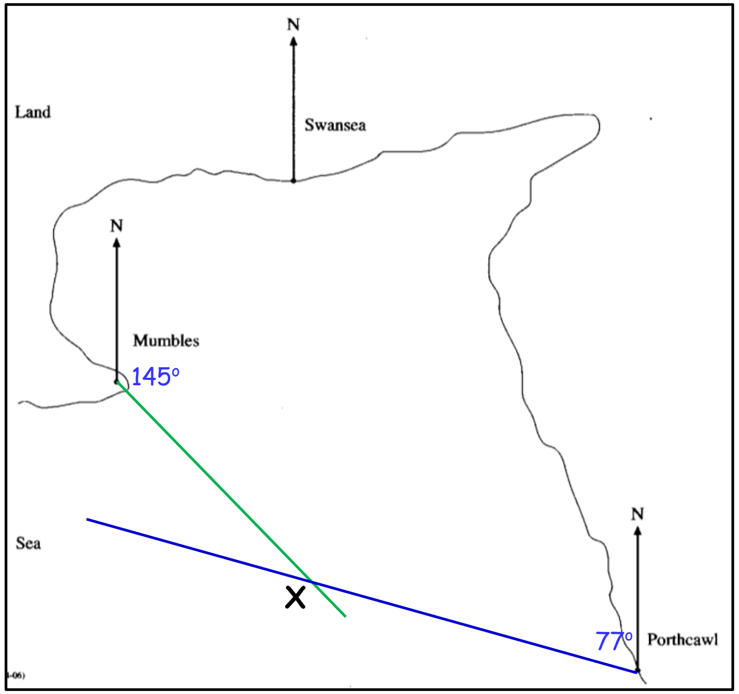


b) A ship is on a bearing of 145° from Mumbles and on a bearing of 283° from Porthcawl. Draw these bearings and mark the position of the ship X.

Measure 145° from Mumbles and draw a straight line going through the angle. You cannot draw an angle of 283° with a normal protractor so subtract from 360° and draw the acute angle anticlockwise.

$$360° - 283° = 77°$$

Draw a straight line through this angle and where the two lines intersect (cross) must be the position of the ship X.



Mathematics

Foundation

Unit 26

Money

Grocery Bills

Example 1: Chris goes shopping. Complete his bill.

Item	Cost
4 litres of milk @ £0.89 per litre	£3.56
6 cartons of apple juice @ £2.47 per carton	£14.82
5 packets of biscuits @ £1.67 per packet	£8.35
3 boxes of tea @ £4.49 per box	£13.47
Total	£36.64

Apple Juice: $6 \times £2.47 = £14.82$

Biscuits: $5 \times £1.67 = £8.35$

Tea: $3 \times £4.49 = £13.47$

Total: $£14.82 + £8.35 + £13.47 = £36.64$

Example 2: Ahmed buys some groceries. Complete the **four** entries in the following table to show his bill for these items.

Amount	Item	Cost (£)
6 packs	Butter @ £1.24 per pack	£7.44
4kg	Sugar @ 86p per kg	£3.44
3 packs	Currants @ £1.54 per pack	4.62
Total		£15.50

Butter: $6 \times £1.24 = £7.44$

Sugar: Be careful, the price of sugar is given in pence not pounds. Either change the pence to pounds first and then work out or work it out as it as and change the answer to pounds (to change pence to pounds divide by 100).

$4 \times £0.86 = £3.44$

Currants: We know the total cost of the currants, the cost per pack, and need to work out how many packs were bought. We need to work backwards.

$£4.62 \div £1.54 = 3$ packs

Total: $£7.44 + £3.44 + £4.62 = £15.50$

Mathematics

Foundation

Unit 26

Household Bills – electricity bills, water bills, etc.

Method

Step 1: Find the number of units used.

Step 2: Calculate the cost of units used, convert to £.

Step 3: Calculate the cost of units used plus the service charge.

Step 4: Calculate the cost of the VAT.

Step 5: Add the cost of VAT on to the total amount.

Example: Ruth gets her electricity bill for the 3-month period July – September 2000.
The details are as follows:

Previous meter reading	46583
Present meter reading	49468
Charge per unit	6.65 pence per unit
Service charge	£10.56
VAT	5%

Write out the details of the cost of electricity for this period and find the total bill including VAT

Step 1: Units used = 49468 – 46583

= 2885 units

Step 2: Cost of units used = 6.65 x 2885

= 19185.25p

Convert to pounds: 19185.25 ÷ 100 = £191.8525

Step 3: Cost of units used plus service charge = £191.8525 + £10.56

= £202.4125

Step 4: 5% VAT 10% = £20.24125

5% = £10.120625

Step 5: Total cost including 5% VAT

202.4125 + 10.120625 = £212.533125

= £212.53 (2 d.p.)

Mathematics

Foundation

Unit 26

Exchange rates

Method

- To convert from British pounds to a **new** currency, you multiply by the exchange rate.

e.g. The exchange rate is £1 = $2.65

So, £90 in dollars would be 90 x 2.65 = $238.50.

- To convert from a **new** currency to British pounds, you divide by the exchange rate.

e.g. The exchange rate is £1 = 1.21€

So 34.50€ in British pounds would be 34.50 ÷ 1.21 = 28.51€ (to 2 d.p.)

Example 2:

Mena goes on holiday to France.
She takes 590 euros with her on holiday.

Mena only spends 40% of her euros.

When she returns from holiday, she exchanges her remaining euros for pounds.
The exchange rate is £1 = 1.18 euros.
How many pounds does Mena receive?

Mena brought 60% of her euros back: 60% of 590 euros = 0.6 x 590

=354 euros

354 euros in pounds: 354 ÷ 1.18 = £300

Example 1:

Ewan is going on holiday to India.
He has saved £450 to exchange for Indian rupees.

(a) The exchange rate on the internet last week was £1 = 99.40 rupees.
Had Ewan been going on holiday last week, how many rupees could he have bought?

450 x 99.4 = 44730 rupees

(b) Ewan exchanges his money on arrival in India.
The exchange rate is now £1 = 99.72 rupees.

The exchange bureau only has 500 rupee notes.
Ewan wants to buy as many rupees as possible with his £450 savings.

How much of his £450 will Ewan spend buying rupees?
Give your answer correct to the nearest penny.
You must show all your working.

With his money Ewan could get: 450 x 99.72 = 44874 rupees

As the bureau only has 500-rupee notes, the most rupees Ewan can have is 44500 rupees (he couldn't have 4500 rupees as he doesn't have enough pounds to exchange)

44500 rupees in pounds is: 44500 ÷ 99.72 = £446.2494…….

So, to the nearest penny, Ewan will spend £446.25 buying rupees

Mathematics

Foundation

Unit 26

Best Buys

Method

- Decide how you are going to compare the offers, how many items or the mass/capacity/cost.
- Use division to get the number of items/capacities that you are going to compare.

Example 1:

Small bottle 300 ml for 66p

Medium bottle 400 ml for 92p

Large bottle 500 ml for £1.25

Roland is going to buy some orange juice for a party.
Which size bottle of orange juice offers the best value for money?
You must show your working.

Compare the capacity (100ml of each)

Small bottle: 300ml is 66p

÷3 ÷3

100ml is 22p

Medium bottle: 400ml is 92p

÷4 ÷4

100ml is 23p

Large bottle: 500ml is £1.25 = 125p

÷5 ÷5

100ml is 25p

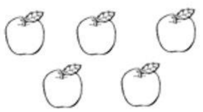
The best value for money the small bottle at 22p per 100ml.

Example 2:

1) Two shops, Kwik Stores and Bob's Fruit and Veg, both sell Pink Lady apples.

Kwik Stores

Pink Lady apples

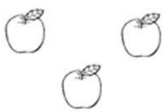


5 for £1.80

At which shop are Pink Lady apples the better value for money? Show all your working.

Bob's Fruit and Veg

Pink Lady apples



3 for £1.05

Compare the cost of 1 apple

Kwik Stores: 5 for £1.80

÷5 ÷5

1 for £0.36

Bob's Fruit and Veg: 3 for £1.05

÷3 ÷3

1 for £0.35

Bob's Fruit and Veg is cheaper by 1p per apple.

Mathematics

Foundation

Unit 26

Income tax

Key Words

Gross income: Money earned (salaries, bonuses etc.)
Taxable income: Money that can be taxed
Personal allowance: Money you don't have to pay tax on
Tax: A compulsory financial charge to fund government expenditures.
Per annum: Per year (annually, yearly etc.)

Method:

Step 1: Draw a diagram. Use it to calculate how much tax is payable from each tax bracket (no tax, 20%, 40%).

Step 2: Calculate the tax due in each tax bracket.

Step 3: Add the together the calculated tax values.

Step 4: Re-read the question, is it asking for annual wage

Example 1: David earns £21,000 per annum. He pays tax at 20% on any earnings over £12,500 per year. Calculate the amount of money he receives after tax each month.

Step 1:

£12500 no tax	£7500 taxed at 20%

Step 2: The tax payable at 20%

20% of £7500:

10% = £750

20% = £1500

David gets £21000 - £1500 = £19500 per annum after tax

Example 2: Claudia was given the following information:

UK Income Tax

April 2013 to April 2014

taxable income = gross income – personal allowance

- personal allowance is £9205
- basic rate of tax: 20% on the first £32 255 of taxable income
- higher rate tax: 40% is payable on all taxable income over £32 255

During the tax year 2013 to 2014, Claudia's gross income was £52 250.
Calculate the total amount of tax that Claudia should pay.
You must show all your working.

Step 1: How much income is taxable?

52250 – 9250 = £43045

£9250 No tax personal	£32255 at 20%	43045 - 32255 = £10790 £10790 at 40%

Step 2: Total tax to be paid at 20%

20% of £32225: 0.2 x 32255 = £6451

Total tax to be paid at 40%

40% of £10790: 0.4 x 10790 = £4316

Step 3: Total amount of tax payable = £6415 + £4316

= £10767

Mathematics

Foundation

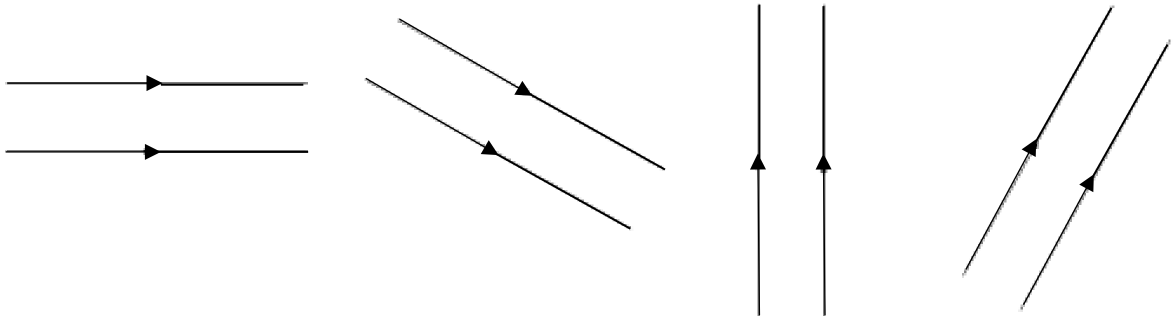
Unit 27

Angles in Parallel Lines

Parallel Lines

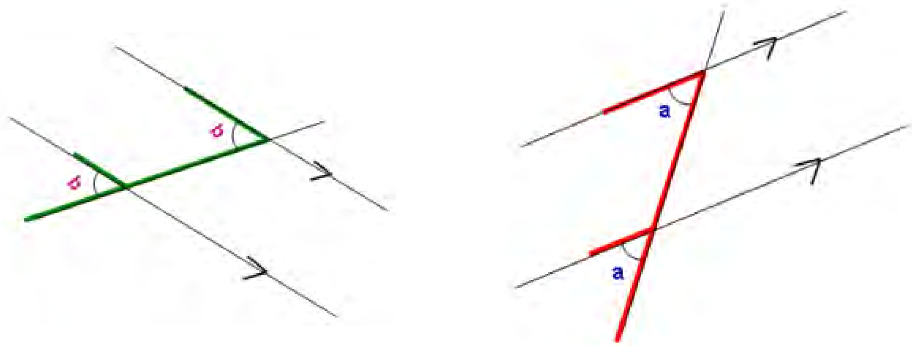
Parallel lines are lines which never meet, and always keep a perfectly equal distance apart.

Remember: Lines are only parallel if they have the little arrows on them.



Corresponding Angles ("F" angles)

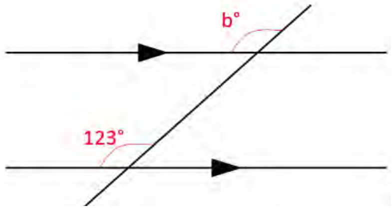
Fact: Corresponding Angles are equal

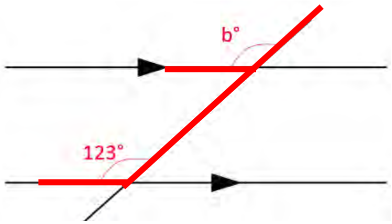


How to spot it: Look for the **F** shape, the angles underneath the arms of the F are equal

Note: The arms of the F must definitely be Parallel lines!

Example: Find the size of angle b.





There is an "F" shape (upside down "F" shape), and both the angles are underneath the arms of the "F".

This means that the two angles are corresponding angles and are equal.

So, $b = 123°$

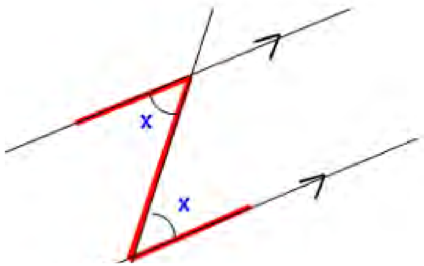
Mathematics

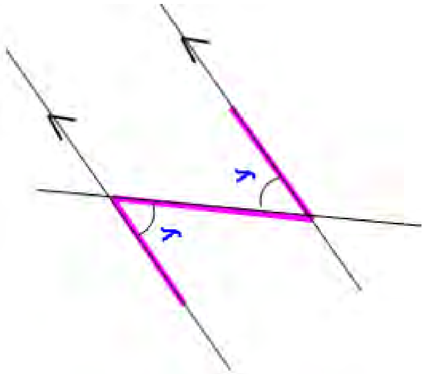
Foundation

Unit 27

Alternate Angles ("Z" angles)

Fact: Alternate Angles are equal

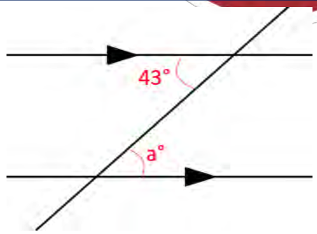


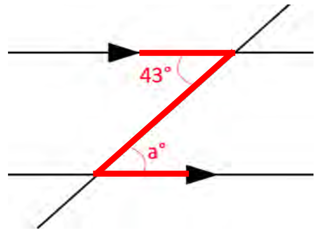


How to spot it: Look for the **Z** shape, the angles "inside" the Z are equal

Note: The top and bottom of the Z must be Parallel Lines!

Example: Find the size of angle a.





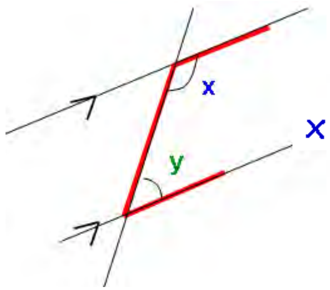
There is an "Z" shape, and both the angles are inside the arms of the "Z".

This means that the two angles are alternate angles and are equal.

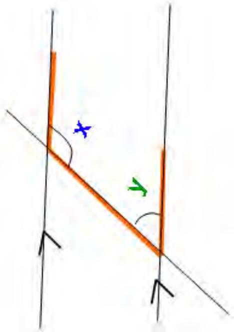
So, $a = 43°$

Interior Angles ("C" or "U" angles)

Fact: Interior Angles add up to 180^0



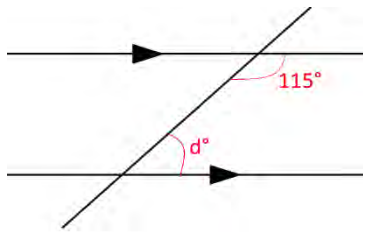
$x + y = 180^0$

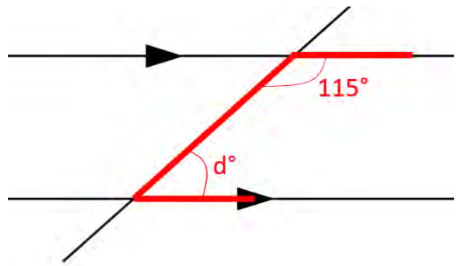


How to spot it: Look for the ***C*** shape, the angles underneath the top and bottom of the *C* add up to 180^0

Note: The top and bottom of the *C* must definitely be Parallel lines!

Example: Find the size of angle d.





There is an "C" shape, and both the angles are inside the "C".

This means that the two angles are alternate angles and add up to 180°.

So, $d = 180 - 115 = 65°$

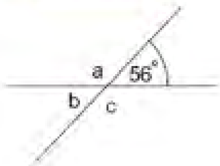
Mathematics

Foundation

Unit 27

Example Questions – Mixed Rules

Example 1

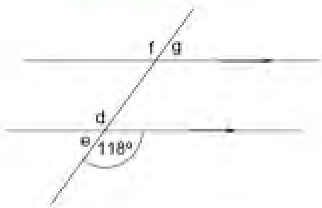


a = 180 - 56 = 124°
(Fact 1 - angles on a straight line)

b = 56°
(Fact 5 - opposite angles)

c = 360 - 56 - 124 - 56 = 124°
(Fact 2 - angles around a point)

Example 2



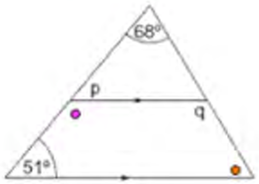
d = 118°
(Fact 5 - opposite angles)

e = 180 - 118 = 62°
(Fact 1 - angles on a straight line)

f = 118°
(Fact 6 - corresponding angles)

g = 180 - 118 = 62°
(Fact 1 - angles on a straight line)

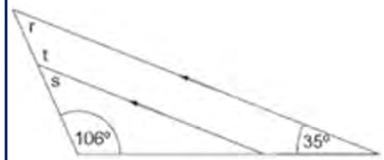
Example 3



p = 51°
(Fact 6 - corresponding angles)

To work out q:
● = 180 – 51 = 129° (Fact 1 – angles on a straight line)
● = 180 – 51 – 68 = 61° (Fact 3 – angles in a triangle)
q = 360 – 51 – 129 – 61 = 119°
(Fact 4 -angles in a quadrilateral)

Example 4

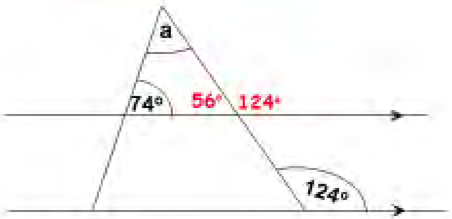


r = 180 – 106 – 35 = 39°
(Fact 3 – angles in a triangle)

s = 39°
(Fact 6 – corresponding angles)

t = 180 – 39 = 141°
(Fact 8 – interior angles)

Example 5

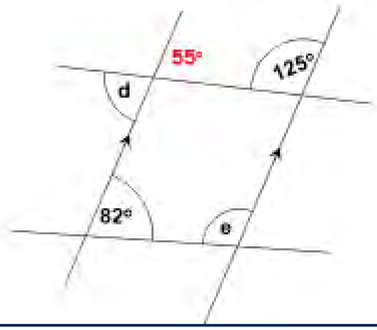


124 (Fact 6 - corresponding)

180 – 124 = 56° (Fact 1 – angles on a straight line)

a = 180 – (74 + 56) = 50° (Fact 3 – angles in a triangle)

Example 6



180 – 125 = 55° (Fact 8 - interior)

d = 55° (Fact 5 – opposite angles)

e = 180 – 82
= 98° (Fact 8 – interior angles)

Mathematics

Foundation

Unit 28

Angles in Polygons

Key Words:

Polygon: The general term for a shape with any amount of sides.

Regular: A shape where all angles and sides are equal.

Irregular: A shape where the sides and angles are not all equal.

Interior Angles: The angles inside a shape.

Exterior Angles: The angles outside a shape.

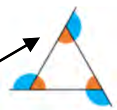
Angles in Polygons Rules

(n is the number of sides in the polygon)

Sum of interior angles = $(n-2) \times 180°$

Sum of exterior angles = 360°

Interior angle + exterior angle = 180°



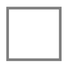
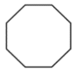
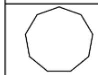
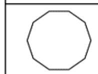
Additional Rules for Angles in a Regular Polygon

One interior angle = Sum of interior angles $\div\, n$

One exterior angle = $360 \div n$

$n = 360 \div$ one exterior angle

Regular Polygons

Shape	Name	Number of sides	Sum of interior angles	One interior angle	Sum of exterior angles	One exterior angle
	Equilateral Triangle	3	$(3-2)\times 180$ $= 180°$	$180 \div 3$ $= 60°$	360°	$360 \div 3$ $= 120°$
	Square	4	$(4-2)\times 180$ $= 360°$	$360 \div 4$ $= 90°$	360°	$360 \div 4$ $= 90°$
	Regular Pentagon	5	$(5-2)\times 180$ $= 540°$	$540 \div 5$ $= 108°$	360°	$360 \div 5$ $= 172°$
	Regular Hexagon	6	$(6-2)\times 180$ $= 720°$	$720 \div 6$ $= 120°$	360°	$360 \div 6$ $= 60°$
	Regular Heptagon	7	$(7-2)\times 180$ $= 900°$	$900 \div 7$ $= 128.6°$	360°	$360 \div 7$ $= 51.4°$
	Regular Octagon	8	$(8-2)\times 180$ $= 1080°$	$1080 \div 8$ $= 135°$	360°	$360 \div 8$ $= 45°$
	Regular Nonagon	9	$(9-2)\times 180$ $= 1260°$	$1260 \div 9$ $= 140°$	360°	$360 \div 9$ $= 40°$
	Regular Decagon	10	$(10-2)\times 180$ $= 1440°$	$1440 \div 10$ $= 144°$	360°	$360 \div 10$ $= 36°$

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Regular Polygon Questions

Finding the Number of Sides of a Regular Polygon

Example 1:

A regular polygon has exterior angles of 30°, how many sides does the polygon have?

Using the rule: $n = 360 \div$ one exterior angle

$n = 360 \div 30$

$n = 12$ The polygon has 12 sides.

Example 2:

A regular polygon has interior angles of 156°, how many sides does the polygon have?

Step 1: Using the rule: Interior angle + exterior angle = 180°

Rearrange to give: Exterior angle = 180 – Interior angle

= 180 – 156

= 24°

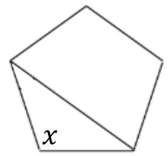
Step 2: Using the rule: $n = 360 \div$ one exterior angle

$n = 360 \div 24$

$n = 15$ The polygon has 15 sides.

Example 3:

The diagram shows a regular pentagon.
Work out the value of x.



Using the rule: Sum of interior angles = $(n-2) \times 180°$

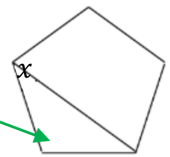
= $(5-2) \times 180°$

= 540°

Using the rule: One interior angle = Sum of interior angles $\div n$

= $540 \div 5$

= 108°



So, $x = 108°$

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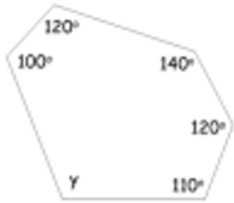
Irregular Polygon Questions

Example 1:

Find the size of angle y.



The shape has 6 sides, so it is a hexagon.

Using the rule: Sum of interior angles = $(n - 2) \times 180°$

= $(6 - 2) \times 180°$

= 720°

Add up the interior angles we already have:

100 + 120 + 140 + 120 + 110 = 600

720 – 600 = 120° $y = 120°$

Example 2:

Four of the interior angles of a seven-sided polygon are 114°, 150°, 160° and 170°. The other three interior angles of this polygon are equal. Calculate the size of each of the other three interior angles.

Using the rule: Sum of interior angles = $(n - 2) \times 180°$

= $(7 - 2) \times 180°$

= 900°

The four interior angles add to: 114 + 150 + 160 + 170 = 584°

900 – 584 = 306°

306 ÷ 3 = 102° Each of the other three interior angles is 102°.

Example 3:

Two of the exterior angles of a hexagon are 110° and 130°. The other exterior angles are all equal. Calculate the size of the largest of the interior angles of this hexagon.

Note: Take care not to get confused, this question talks about exterior angles AND interior angles.

Using the rule: Sum of exterior angles = 360°

360 – (110 + 130) = 120°

120 ÷ 4 = 30°

The other interior angles are all 30°

Note: The smallest exterior angles will give the largest interior angles.

Using the rule: Interior angle + exterior angle = 180°

Rearrange to give: Interior angle = 180 – Exterior angle

= 180 – 30

= 150°

The largest of the interior angles is 150°.

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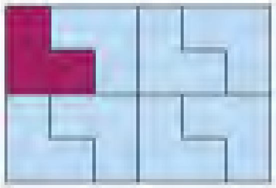
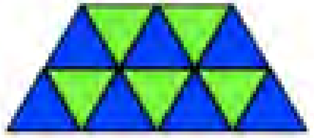
Tessellation

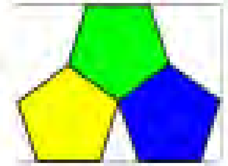
A tessellation is a pattern created with identical shapes that fit together with no gaps.

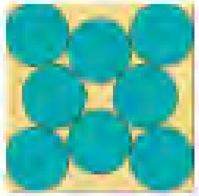
These shapes tessellate – they fit together with no gaps between them.





These shapes do not tessellate – when they are put together, they have gaps between them.

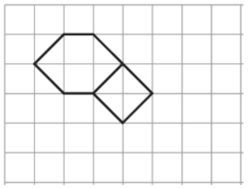


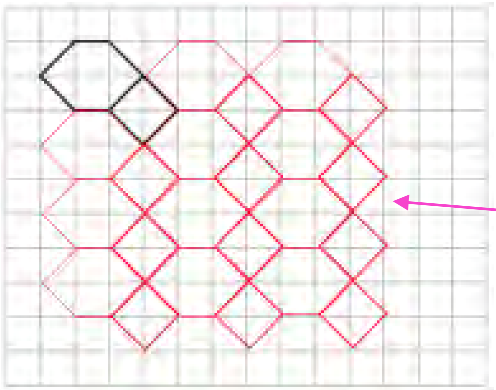


Regular polygons tessellate if the interior angles can be added together to make 360° (a full turn), i.e. if one interior angle is a factor of 360.

Example 1:

Ben needs to tile his kitchen floor and decides to use the two types of tiles shown in the diagram. By drawing more tiles in the diagram, show that the tiles will tessellate.

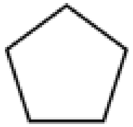




The shapes fit together with no gaps.

Example 2:

Shown is a regular pentagon. Will the regular pentagon tessellate? You must show your workings.



Using the rule: Sum of interior angles = $(n-2) \times 180°$

= $(5-2) \times 180°$

= 540°

Using the rule: One interior angle = Sum of interior angles $\div n$

= $540 \div 5$

= 108°

Is 108° a factor of 360? $\frac{360}{108} = 3.3$

108° is not a factor of 360°, therefore a regular pentagon will not tessellate.

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Constructing and Interpreting Graphs in Everyday Life

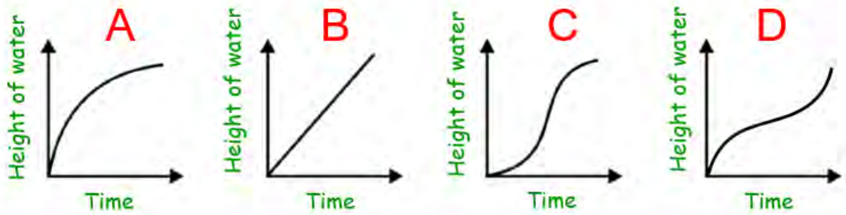
Often you will be presented with a "real life" graph and asked a few questions based upon it.

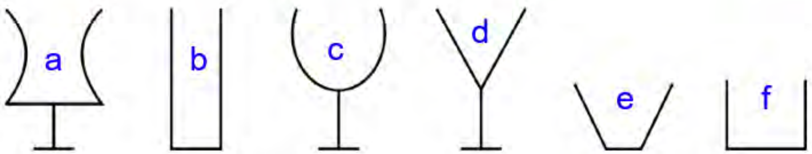
Method for Interpreting Real-Life Graphs

- Look carefully at both **axes** to see what the variables are
- Look at the **scale** carefully so you can accurately read the graph
- Look at the **gradient** of the graph:
  What does a horizontal line mean?
  What does a positive/negative slope mean?
- Always **read** the question carefully and **check** your answers.

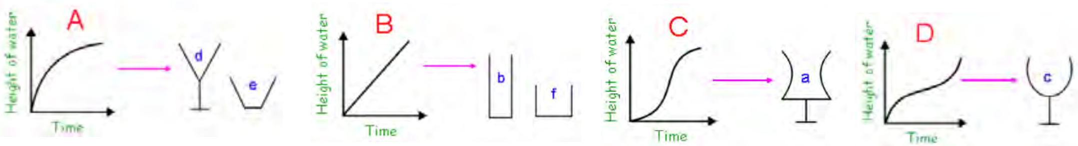
Example – Story Graph

Water is poured into various glasses at a constant rate. The graphs below are sketches showing how the height of water in the glasses' changes over time. Match up the shape of the glasses with their graphs
Note: Each graph can represent more than one glass.





Answers:



- Look carefully at both **axes** to see what the variables are
  We have height of water going up the y-axis, and time going along the x-axis
- Look at the **scale** carefully so you can accurately read the graph
  There is no scale, so this doesn't apply
  Note: This is also the reason why more than one glass can match to each graph
- Look at the **gradient** of the graph:
  What does a **straight**-line mean?
  The height of the water is changing by the same amount as time passes, so the sides of the glass must be straight!

  What does a **curved** line mean?
  Well, it depends on the shape of the curve, but generally a curved line means that the height of the water is not changing by the same amount, so the sides of the glass must also be curved
- Try to picture that water dropping constantly into those glasses and what the height of the water will be doing.

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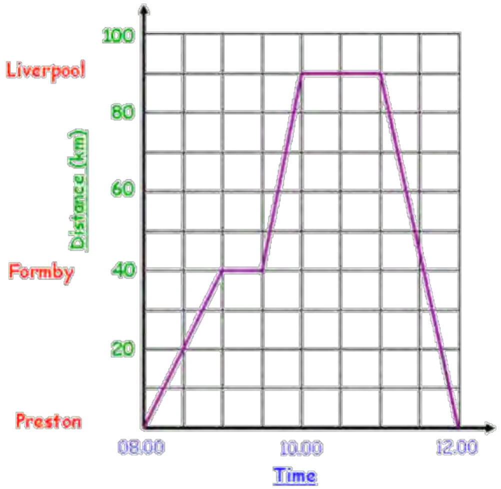
Example – Travel Graph



The graph on the left shows a journey made by a family in a car between Preston, Formby and Liverpool. Look at the graph and then answer the following questions:

(a) What time did the family arrive in Liverpool?

(b) What is the distance from Formby to Liverpool?

(c) How long did the family spend not moving?

(d) What was the average speed on the journey home?

- Look carefully at both **axes** to see what the variables are
  We have distance in kilometres going up the y-axis, and time in hours going along the x-axis
- Look at the **scale** carefully so you can accurately read the graph
  On the y-axis every square represents 10km, and on the x-axis every square is 15 minutes
- Look at the **gradient** of the graph
  What does a **horizontal** line mean?
  A horizontal line means that time is still passing, but the distance travelled is not changing, so the family must have stopped moving.

  What does a **positive/negative slope** mean?
  A positive slope means the family are travelling from Preston towards Liverpool, and a negative slope means they are on their way back home.

  Note: You could say that the family are travelling faster between Formby and Liverpool than between Preston and Formby, we know this because the line is steeper meaning they are travelling more distance in less time, so they must be going faster.
- We can now answer all the questions.

Answers:

(a) What time did the family arrive in Liverpool?

The line first hits Liverpool at 10.00

(b) What is the distance from Formby to Liverpool?

Formby is 40km from Preston, Liverpool is 90km from Preston, so the distance from Formby to Liverpool must be 50km.

(c) How long did the family spend not moving?

When the family is not moving we see a horizontal line. That happens twice, firstly at Formby for 30 minutes, and then at Liverpool for 60 minutes, giving us a total of 90 minutes, or one and a half hours.

(d) What was the average speed on the journey home?

Using the formula: Speed = Distance ÷ Time

On the journey home we have: Speed = 90km ÷ 1 hour

= 90 km/hr

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Example – Conversion Graph

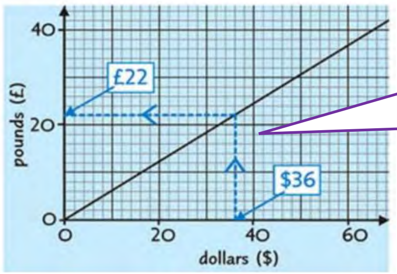
A conversion graph is used to change one unit into another.

This could be changing between miles and kilometres, pounds to a foreign currency, or the cost of a journey based on the number of miles travelled.

Method for using conversion graphs:

1. Draw a line from a value on one axis – keep going until you hit the line.
2. Change direction and go straight to the other axis – the value you get on this axis is equivalent (the same as) to the value on the other

Example: Doug went on holiday to South Carolina and paid **\$360** for a PlayStation. On the way back Doug saw the same PlayStation in Cardiff Airport for **£250**. Did Doug get a good deal while on holiday?



Make sure you draw your conversion lines on the graph – these are your **workings**.

Answering the question:

\$360 dollars isn't on the graph, so you need to find a way of making the calculation as easy as possible for yourself. In this question the easiest way is to read off the value for \$36 and then multiply by 10 (because \$36 x 10 = £360).

Reading off the graph: \$36 = £22

So \$360 would be: £22 x 10 = £220

To finish you need to compare the values and add a conclusion:

£220 is less than £250, so Doug got the best deal as the PlayStation was cheapest in South Carolina.

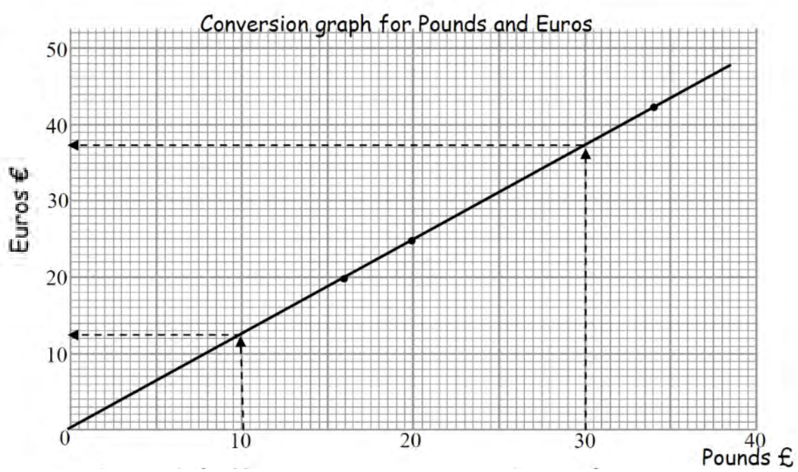
Method to draw a Conversion Graph

- For a conversion graph you need at least 3 pairs of values that are equivalent to each other. Eg one pair could be 1 inch ≡ 2.54 cm
- Decide on the scale you are going to use for the 1st set of data. This is usually on the horizontal axis.
- Decide on the scale you are going to use for the 2nd set of data. This is usually on the vertical axis.
- The vertical axis does not have to have the same scale as the horizontal axis but each axis must have a "uniform scale".
- Each axis should start from zero.
- The values are placed on the lines not in the spaces.
- Complete both axes and label fully.
- Plot each point by reading across to its horizontal value and up to its corresponding vertical value. Mark the position with either a cross or a dot.
- Once all the points have been plotted join them up with a straight line that **passes through all** the points.
- The conversion graph can then be used to answer questions such as converting from one value to another.
- Write a title for your conversion graph.

Example

(a) Draw a conversion graph for

Pounds £	16	20	34
Euros €	20	25	42.50



b) Using the graph, find how many Euros are equivalent to £30.

€ 37.50

c) How many Euros are equivalent to £50? Explain fully how you got your answer.

Find how many Euros are equivalent to £10 and then multiply by 5

£10 = €12.50

£50 = 12.50 x 5 = € 62.50

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Solving Equations 2

A linear equation is an equation (has an equals sign) involving letters and numbers, where the highest power of any letter is 1.
The aim of solving an equation is to find the value of the unknown which makes the equation balance, e.g. equation: $x - 5 = 3$, solution: $x = 8$, because $8 - 5 = 3$.

There are different methods you can use to solve equations using your knowledge of inverse operations.

An operation is a mathematical process such as adding, multiplying, or squaring, etc.

An inverse operation is the process of reversing the operation (the opposite process). For example, when adding, the inverse operation would be subtracting, when multiplying the inverse operation would be division and so on.

Here are the main inverse operations you need to know:

$+ \rightleftarrows -$

Addition is the opposite of subtracting.
Subtracting is the opposite of adding.
They are inverse operations.

$\times \rightleftarrows \div$

Multiplication is the opposite of division. Division is the opposite of multiplication
They are inverse operations.

Method 1: Using our knowledge of inverse operations we can rearrange the equation to get the letter (this is often x) on its own. Many teachers say this is called the "Change the side, change the sign" method.

Golden Rule: When rearranging an equation and moving a term over the equals sign to the **opposite side** it changes to the **opposite sign** (the inverse). For example, '+3' becomes '-3', or '÷4' become 'x4'.

Note: The subject term is the letter used in the equation.

Step 1: Get rid of any square root signs by squaring both sides. Clear any fractions by cross-multiplying up to every other term. Multiply out any brackets.

Step 2: Collect all subject terms on one side of the equals sign and all non-subject terms on the other. Remembering the rule "change sides, change sign" (you most often see the letters on the left-hand side and numbers on the right).

Step 3: Simplify like terms on each side of the equation.

Step 4: If you are left with a number multiplied by your subject term equals something ($Ax = B$ where A and B are numbers and x is the subject term), then to get the subject term on its own, move the number over the other side of the equals sign remembering to change its sign to the opposite sign (the inverse) which in this case is from a multiply to a divide ($Ax = B$ becomes $x = \frac{B}{A}$).

Check your answer using substitution to make sure you are right.

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Example 1: $7p - 3 = 32$

$$7p - 3 = 32$$

Move the -3 over the equals sign to the opposite side and change the sign to the opposite / inverse operation

$$7p = 32 + 3$$

Remember, $7p$ means $7 \times p$

$$7p = 35$$

Move the $\times 7$ over the equals sign to the opposite side and change the sign to the opposite / inverse operation

$$p = \frac{35}{7}$$

Remember, $\frac{35}{7}$ means $35 \div 7$

$$p = 5$$

Example 2: $2(3r + 6) = 36$

$$2(3r + 6) = 36$$

Expand the bracket first:

$$6r + 12 = 36$$

Move the $+12$ over the equals sign to the opposite side and change the sign to the opposite / inverse operation

$$6r = 36 - 12$$

$$6r = 24$$

Move the $\times 6$ over the equals sign to the opposite side and change the sign to the opposite / inverse operation

$$r = 24 \div 6$$

$$r = 4$$

Example 3: $6 + \frac{k}{5} = -1$

Remember, if there is no sign in front it means it is a plus

$$6 + \frac{k}{5} = -1$$

Move the $+6$ over the equals sign to the opposite side and change the sign to the opposite / inverse operation

$$\frac{k}{5} = -1 - 6$$

$$\frac{k}{5} = -7$$

Move the $\div 5$ over the equals sign to the opposite side and change the sign to the opposite / inverse operation

$$k = -7 \times 5$$

$$k = -35$$

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Example 4: $24 - 3m = 6$

$$24 - 3m = 6$$

Move the $+24$ over the equals sign to the opposite side and change the sign to the opposite / inverse operation

$$-3m = 6 - 24$$

$$-3m = -18$$

Remember, even though it is a – 3, it is being multiplied by the m, so the opposite / inverse operation is a divide

Move the $\times\,(-3)$ over the equals sign to the opposite side and change the sign to the opposite / inverse operation

$$m = \frac{-18}{-3}$$

$$m = 6$$

Example 5: $7y + 3 = 10y - 6$

$$7y + 3 = 10y - 6$$

Move the $+3$ over the equals sign to the opposite side and change the sign to the opposite / inverse operation

$$7y = 10y - 6 - 3$$

$$7y = 10y - 9$$

Move the $+10y$ over the equals sign to the opposite side and change the sign to the opposite / inverse operation

$$7y - 10y = -9$$

$$-3y = -9$$

Move the $\times\,(-3)$ over the equals sign to the opposite side and change the sign to the opposite / inverse operation

$$y = \frac{-9}{-3}$$

$$y = 3$$

Example 6: $5(x - 3) = 4(x + 2)$

$$5(x - 3) = 4(x + 2)$$

Expand the brackets on both sides

$$5x - 15 = 4x + 8$$

Move the -15 over the equals sign to the opposite side and change the sign to the opposite / inverse operation

$$5x = 4x + 8 + 15$$

$$5x = 4x + 23$$

Move the $+4x$ over the equals sign to the opposite side and change the sign to the opposite / inverse operation

$$5x - 4x = 23$$

$$x = 23$$

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Method 2: Balancing equations

Golden Rule: Whatever you do to one side of the equation, you must do exactly the same to the other side to keep the equation in balance

Step 1: If they are not already, get all your unknown letters on one side of the equation (NOT on the bottom of fractions and avoiding negatives).

Step 2: Begin undoing the operations that were done to your unknown letter, by thinking about the order that things were done to the letter

Step 3: Use inverse operations to do this until you are left with just your unknown letter on one side, and the answer on the other

Step 4: Check your answer using substitution to make sure your answer is right.

Example 1: $7p - 3 = 32$

Step 1: The unknown letter (p) only appears on the left-hand side of the equation, there is no negative sign in front of it, and it is not on the bottom of a fraction.

$$7p - 3 = 32$$

Step 2: What order were things done to p? First it was multiplied by the 7, then 3 was subtracted.

Step 3: To undo the operations, we start with the last one, working our way backward and apply the inverse (opposite) operation to both sides:

The last operation was -3, so the opposite / inverse operation is +3, remembering the rule whatever you do to one side of the equation you do to the other.

Now divide both sides by 7

$$7p - 3 = 32$$
$$+3 \qquad\qquad +3$$
$$7p = 35$$
$$\div 7 \qquad\qquad \div 7$$
$$p = 5$$

Step 4: Check if the answer is right. Substitute $p = 5$ into the initial equation.

When p = 5

$$7p - 3 = 7 \times 5 - 3 = 35 - 3 = 32$$

Example 2: $24 - 3m = 6$

Step 1: The unknown letter (m) only appears on the left-hand side of the equation, it's not on the bottom of a fraction, but it does have a negative sign in front of it.

We can use inverse operations to cancel out the -3m, we just need to add 3m to both sides.

$$24 - 3m = 6$$
$$+3m \qquad\qquad +3m$$
$$24 = 6 + 3m$$

Step 2: What order were things done to m? First it was multiplied by the 3, then 6 was added.

Step 3: To undo the operations, we start with the last one, working our way backward and apply the inverse (opposite) operation to both sides:

The last operation was +6, so the opposite / inverse operation is -6, remembering the rule whatever you do to one side of the equation you do to the other.

Now divide both sides by 3

$$-6 \qquad\qquad -6$$
$$18 = 3m$$
$$\div 3 \qquad\qquad \div 3$$
$$6 = m \quad \text{or} \quad m = 6$$

Step 4: Check if the answer is right. Substitute $m = 6$ into the initial equation.

When m = 6

$$24 - 3m = 24 - 3 \times 6 = 24 - 18 = 6$$

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Example 3: $6+\frac{k}{5} = -1$

$$6+\frac{k}{5} = -1$$

$$-6 \qquad\qquad -6$$

$$\frac{k}{5} = -7$$

$$\times 5 \qquad\qquad \times 5$$

$$k = -35$$

Check: Substitute k= -35 into the original equation.

$6+\frac{k}{5} = 6+\frac{-35}{5} = 6+-7 = 6-7 = -1$

Example 4: $2(3r+6) = 36$

$$2(3r+6) = 36$$

Expand brackets

$$6r+12 = 36$$

$$-12 \qquad\qquad -12$$

$$6r = 24$$

$$\div 6 \qquad\qquad \div 6$$

$$r = 4$$

Check: Substitute $r = 4$ into the original equation.

$2(3r+6) = 2(3\times 4+6) = 2(12+6) = 2\times 18 = 36$

Example 5: $7y+3 = 10y-6$

$$7y+3 = 10y-6$$

$$-7y \qquad\qquad -7y$$

$$3 = 3y-6$$

$$+6 \qquad\qquad +6$$

$$9 = 3y$$

$$\div 3 \qquad\qquad \div 3$$

$3 = y$ or $y = 3$

Check: $10y-6 = 10\times 3-6 = 24$

Example 6: $5(x-3) = 4(x+2)$

$$5(x-3) = 4(x+2)$$

expand $\qquad$ expand

$$5x-15 = 4x+8$$

$$-4x \qquad\qquad -4x$$

$$x-15 = 8$$

$$+15 \qquad\qquad +15$$

$$x = 23$$

Check: Substitute $c = 23$ into the original equation.

$$5(23-3) = 4(23+2)$$

$$5\times 20 = 4\times 25$$

$$100 = 100$$

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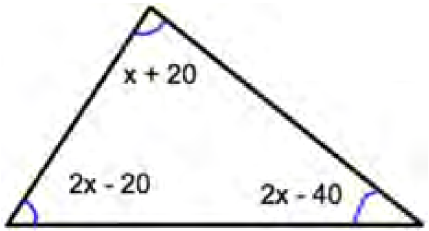
Forming and Solving Equations

Sometimes we are given information and need to form an equation using the information, before solving the equation.

Example 1: The angles in the triangle are $(x+20)°$, $(2x-20)°$, and $(2x-40)°$. Form an equation and use it to find the value of x.

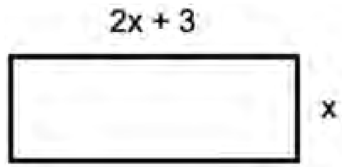


Angles in a triangle add to 180°, so $(x+20)$ plus $(2x-20)$ plus $(2x-40)$ is equal to 180.

$$x + 20 + 2x - 20 + 2x - 40 = 180$$
$$5x - 40 = 180$$
$$5x = 220$$
$$x = 44°$$

Example 2: The perimeter of the rectangle is 42cm.

a) Form an equation in x and solve it to find the value of x.



The perimeter is the distance all the way around the shape, so a length $(2x+3)$ plus a width (x) plus another length $(2x+3)$ plus another width (x) is equal to 42cm.

$$2x + 3 + x + 2x + 3 + x = 42$$
$$6x + 6 = 42$$
$$6x = 36$$
$$x = 6cm$$

b) Calculate the area of the rectangle.

The length of the rectangle is $2 \times 6 + 3 = 15cm$

The width of the rectangle is $6cm$

So, the area of the rectangle is $15 \times 6 = 90cm^2$

Example 3: Jane is 4 years older than Tom.

David is twice as old as Jane.

The sum of their three ages is 60.

Form an equation and use it to find the age of each person.

Let Tom's age $= x$

Jane's age $= x + 4$

David's age is $2(x+4) = 2x + 8$

The sum of their ages is 60:

$$x + x + 4 + 2x + 8 = 60$$
$$4x + 12 = 60$$
$$4x = 48$$
$$x = 12$$

So, Tom is 12 years old

Jane is $12 + 4 = 16$ years old

David is $2 \times 12 + 8 = 32$ years old.

Mathematics

Foundation

Unit 31

Scatter Diagrams

A **Scatter Diagram** shows the relationship between two variables. **Correlation** is used to describe the relationships.

Remember: When choosing a scale, make sure you always go up in **equal steps** along each axis.

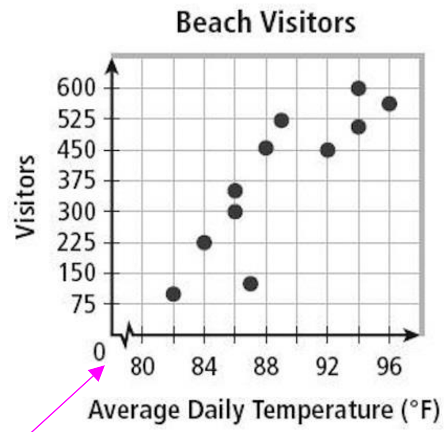
Drawing a Scatter Diagram

Method

- Decide on the scale you are going to use for the 1st set of data. This is usually on the horizontal axis.
- Decide on the scale you are going to use for the 2nd set of data. This is usually on the vertical axis.

  (Note: It does not really matter which set of data goes on the x axis and which on the y;
  I would recommend putting the one with the biggest numbers on the y axis.
  Remember to label both axes, including units.

- The vertical axis does not have to have the same scale as the horizontal axis, but each axis must have a 'uniform scale'.
- Each axis does not need to start from zero.
- The values are placed on the lines not in the spaces.
- Complete both axes and do not forget to LABEL fully.
- Plot the points carefully and mark with a dot or cross. Do not join up the points.



This symbol means a chunk has been taken out of the axis – which means it does not have to start at zero.

The Line of Best Fit

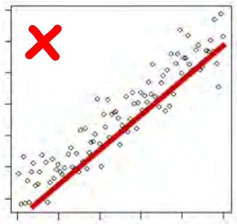
This is a single straight line which is supposed to be a good representation of the pattern / trend of the data.

When drawing the line of best fit:

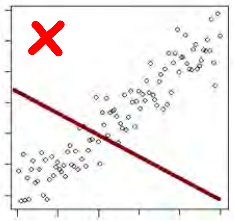
- Make sure the line follows the trend of data.
- Try to get roughly the same amount of points above the line as below
- Experiment by using your ruler as your line, and only draw the line in when you are happy
- Do not spend too long deciding, and do not try to make it perfect.

Note: Your line does NOT have to start at the origin (0, 0)

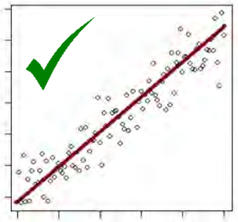
Note: Your line does



A lot more points above than below



Does not follow the trend of data



Good line of best fit

Mathematics

Foundation

Unit 31

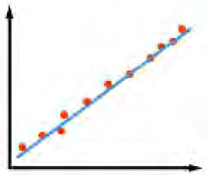
Correlation

The most important use of scatter diagrams is to determine the type (if any) of correlation between two variables

Correlation is the relationship between the two variables.

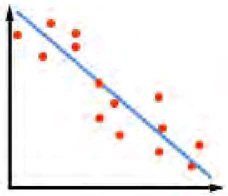
Positive Correlation

As one variable increases, so does the other.



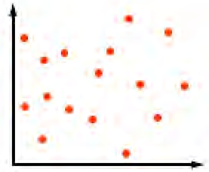
Negative Correlation

As one variable increases, the other decreases.



No Correlation

No relationship between the variables.



It is also worth noting the strength of the correlation.

<u>STRENGTH</u>

Strong – dots are close to each other

Weak – dots are far apart

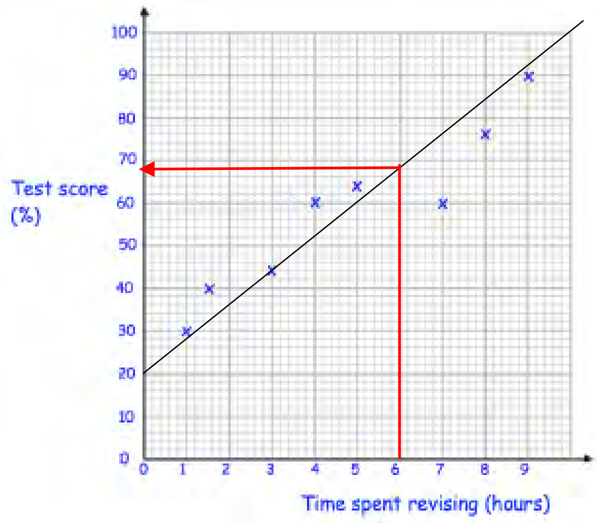
We can use the line of best fit and the correlation to <u>predict results we don't already have.</u>

<u>Note:</u> The <u>stronger</u> the correlation, the <u>more reliable</u> these predictions will be.

Example 1:

Below is a table showing the time each pupil spent revising and the test score they achieved.
Draw a scatter diagram and include the line of best fit.

Time (hours)	1.5	4	8	1	5	9	7	3
Test Score (%)	40	60	76	30	64	90	60	44



a) What type of correlation is shown?

Positive Correlation

b) Another student spent 6 hours revising for the test. Find an estimate of their test score.

Draw a line of best fit and read from it - 68%

c) Explain why it might not be sensible to use the scatter graph to estimate the score for a student that spent 15 hours revising.

It is out of the data range.

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Foundation

Unit 32

Probability 2

Probability is the likelihood that an event will occur.

Probabilities are always written as fractions, decimals, or percentages.

Probabilities have values between 0 and 1.

Probability scale

Probabilities can be described using words

Impossible — Unlikely — Even Chance — Likely — Certain

What is the probability that if you toss a coin it will land on heads?
Even Chance (the coin is equally likely to land on heads or tails)

What is the probability that Winter follows Summer?
Impossible

What is the probability that Christmas will be on the 25th of December?
Certain

Probabilities can also be described using numbers

Impossible	Unlikely	Even	Likely	Certain
0		1/2		1
0		0.5		1.0
0%		50%		100%

The probability of an event happening can be found using:

P(event happening) = $\dfrac{\text{number of ways the event could happen}}{\text{the total number of outcomes}}$

Example: Find the probability of throwing an even number on a dice.

$$\text{P(even number)} = \frac{3}{6}$$

3 — Number of even numbers on a dice (2, 4, 6)

6 — Total amount of numbers on a dice

Example: What is the probability of picking a diamond from a full deck of cards?

$$\text{P(diamond)} = \frac{13}{52}$$

13 — Number of diamonds in a pack of cards

52 — Total number of cards in a pack of cards

The probability of an event **not** happening can be found using:

P(event not happening) = 1 – P(event happening)

Example: What is the probability of **not** picking a diamond from a full deck of cards?

P(not diamond) = 1 – P(diamond)

$$= 1 - \frac{13}{52} \quad = \frac{39}{52}$$

Mathematics

Foundation

Unit 32

Listing Outcomes

You might be asked to list all the possible outcomes for two or more events.

Example: List all the 3-digit numbers that can be made using the digits 3, 6, and 9?

369 396 639 693 936 963

Example: A coin is flipped, and a dice is rolled. List all the possible outcomes.

A head on the coin

A 1 on the dice

H 1 H 2 H 3 H 4 H 5 H 6

A tail on the coin

T 1 T 2 T 3 T 4 T 5 T 6

A 6 on the dice

Sample Space Diagram

A sample space diagram is a way of showing multiple outcomes in one diagram.

Example: Two dice are thrown, and the numbers are multiplied together. The table below shows some of the possible outcomes.

First dice x second dice
5 x 6 = 30

Second Dice						
6	6	12	18	24	30	36
5	5	10	15	20	25	30
4	4	8	12	16	20	24
3	3	6	9	12	15	18
2	2	4	6	8	10	12
1	1	2	3	4	5	6
	1	2	3	4	5	6

First Dice

First dice x second dice
6 x 3 = 18

a) Complete the table to show **all** the possible outcomes.

Number of outcomes that are odd numbers

b) What is the probability of getting an outcome that is an odd number? $P(\text{odd}) = \frac{9}{36} = \frac{1}{4}$

Total number of outcomes

c) If the two dice were thrown a total of 60 times, how many times would you expect to get an outcome greater than 10? $P(6 \text{ and } H) = 60 \times \frac{9}{36} = 15$ times

Number of times the dice are thrown

Probability of an odd number

Finding Missing Probabilities from a Table

Probabilities add up to 1, to find the missing probabilities add together the probabilities you are given and subtract them from 1.

Example: A biased spinner has 4 colours. The probability of the spinner landing on each colour is given below.

Colour	Red	Blue	Yellow	Green
Number of times	0.1	x	0.4	0.2

a) What is the probability of choosing a blue sweet?

Add the probabilities

P(Blue) = 0.1 + 0.4 + 0.2 = 0.7

1 - 0.7 = 0.3

Subtract them from 1

b) The spinner is spun 100 times. Calculate an estimate for the number of times the spinner will land on yellow.

Number of times the spinner is spun

Probability of a yellow

P(Yellow) = 100 x 0.4 = 40 times

Mathematics

Foundation

Unit 33

Transformations

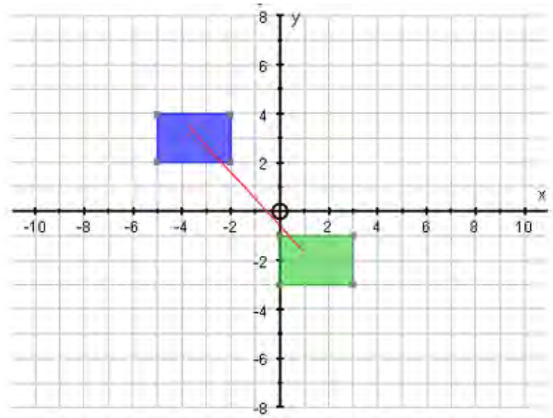
Transformations are specific ways of moving objects, usually around a co-ordinate grid

There are 4 types of transformations you need to know, and for each one you must:

- be able to carry out a transformation yourself
- be able to describe a transformation giving all the required information

Translation

A Translation is a movement in a straight line, it is described by a movement right/left, followed by a movement up/down



If we translate the blue object 5 squares to the right and 5 squares down

We end up with the green object

Describing Translations

Translations can be described using words or vectors.

Example: Translate the object 2 squares to the right and 4 squares down.

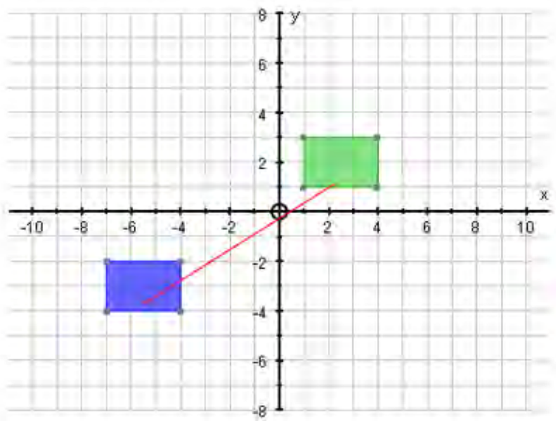
Or

Translate the object using the column vector $\binom{2}{-4}$

$$\begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

If this number is positive you move right, if it is negative you move left.

If this number is positive you move up, if it is negative you move down.



If we translate the blue object by the vector: $\begin{pmatrix} 8 \\ 5 \end{pmatrix}$

8 to the right

5 up

We end up with the green object

Note: If you pick any co-ordinate on the blue shape and translate it by the same vector, you end up with the matching corner on the green shape

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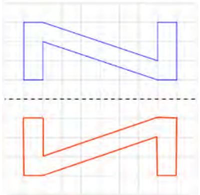
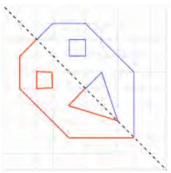
Foundation

Unit 33

Reflection

Reflecting an object across a line produces an exact replica (mirror image) of that object on the other side of the line.



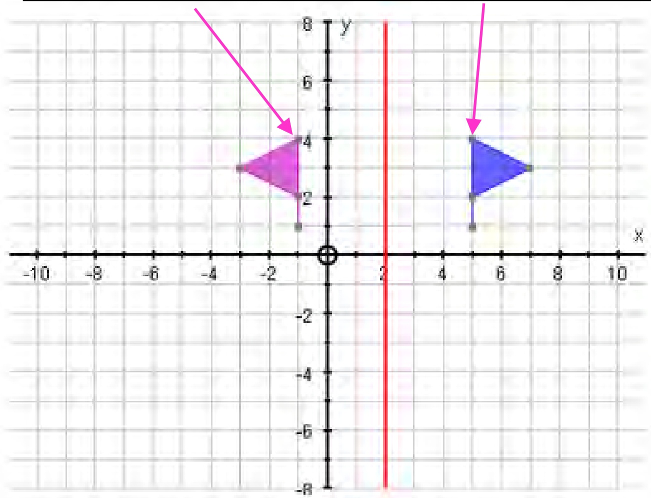


This new shape is called the Image

Describing Reflections

You must give either the equation of the line of reflection (mirror line) or draw the line on the grid.

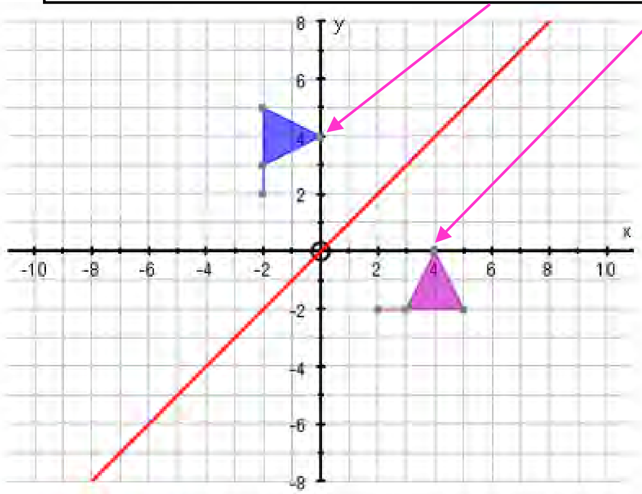
Note: This corner of the blue object is 3 squares from the line, so its corresponding point on the purple object will also be 3 squares from the



If we reflect the blue object in the red line (equation $x = 2$), we end up with the purple object

Note: Every point on the purple object (the image) is the exact same distance from the line of reflection as the matching point on the blue object

Note: This corner of the blue object is 4 squares horizontally from the line, as the line is diagonal its corresponding point on the purple object will be 4 squares vertically from the line



If we reflect the blue object in the red line (equation: $y = x$), we end up with the purple object

Note: Every point on the image is the same distance away from the mirror line as the matching point on the original object.

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Foundation

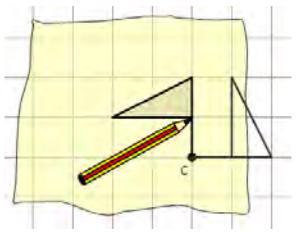
Unit 33

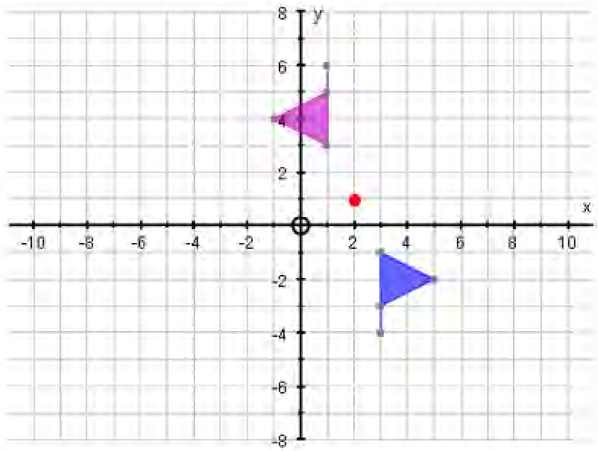
Rotation

Rotating an object means turning the whole shape around a fixed point by a certain number of degrees and in a certain direction.

Remember: If you cannot do these just by looking at the shape, then:

- trace around the object
- place your pencil at the centre of rotation (the fixed point)
- turn the tracing paper around
- draw your rotated object.





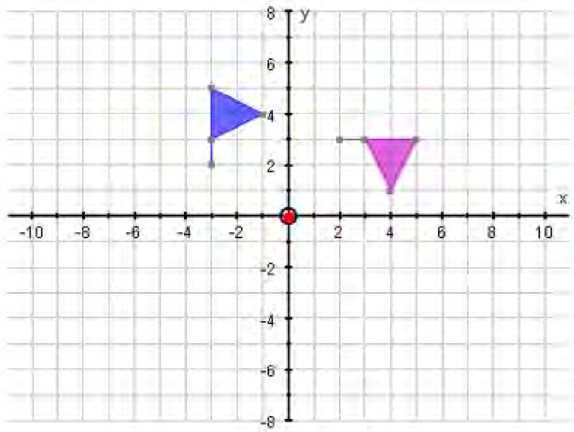
Rotating the blue object 180^0 about the point (2, 1) gives the purple object.

Note: Whenever the angle of rotation is 180^0, it doesn't matter whether you go clockwise or anti-clockwise.

Describing Rotations

You must give all of the following:

1. The **centre** of rotation (give as a co-ordinate if you can)
2. The **direction** of the rotation (clockwise or anti-clockwise)
3. The **angle** of the rotation (usually either 90^0, 180^0 or 270^0)



To describe the rotation from the blue object to the purple object, we would say:

1. Centre of Rotation: (0, 0) (the origin)
2. Direction of Rotation: Clockwise
3. Angle of Rotation: 90^0

Rotate the blue object 90^0 clockwise about the point (0, 0) (or about the origin)

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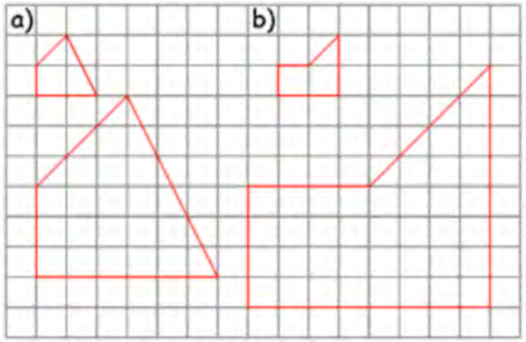
Unit 33

Enlargement

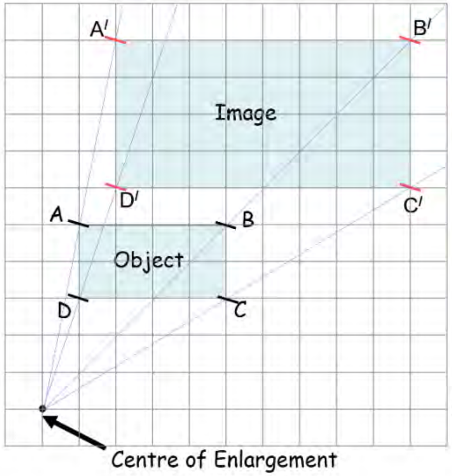
Enlargement is the only one of the four transformations which changes the size of the object

Note: Each length is increased by the same scale factor



(a) The smaller shape in diagram (a) has been enlarged by a scale factor of 3
(b) The smaller shape in diagram (b) has been enlarged by a scale factor of 4

Enlargements from a Given Point



To enlarge the rectangle by scale factor x2 from the point shown

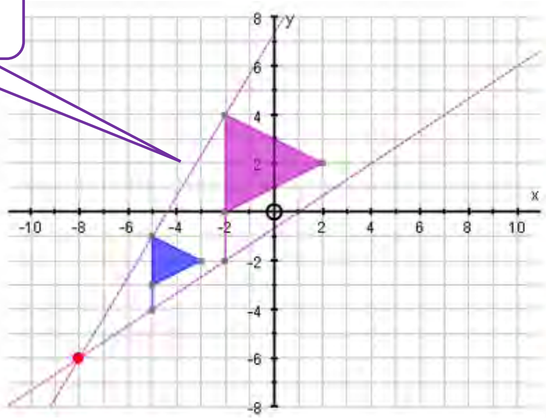
1. Draw the projection lines through corners.

2. Mark off x2 distances along each line.

3. Draw and label image.

Or Count Squares i.e. from the centre: A is 1 across, 5 up so A' is going to be x2 so 2 across and 10 up.

These are called projection lines.



To describe the enlargement from the blue object to the purple object, we would say:

1. Centre of Enlargement: (-8, -6)

2. Scale Factor of Enlargement: 2

Note:

(1) To find the centre of enlargement you must draw line through matching points on both objects and see where they cross

(2) Each point on the purple object is twice as far away from the centre of enlargement than the matching point on the blue.

Describing Enlargements

To fully describe an enlargement, you must give:

1. The centre of enlargement (give as a co-ordinate if you can)
2. The scale factor of the enlargement

Mathematics

Foundation

Unit 34

Questionnaires

Questionnaires or surveys are used to gather data.
You will be required to design or criticise questions on questionnaires.

What to look out for in questionnaires:

Overlapping interval

☐ 0 – 1 hours ☐ 1 – 2 hours ☐ 3 – 4 hours

Which box do you tick if your answer is one hour?

No time scale given

How much time do you spend playing sport?

Is this how much time spent per day, per week…?

Not all options covered by the tick boxes

1-5 times ☐ *6-10 times* ☐ *10 or more times* ☐

What if your answer is 0 hours?

Irrelevant questions

Q1. What is your address?

Do you need to know the persons address?

Biased questionnaires

Martha wants to test the following hypothesis.

'More men than women buy a daily newspaper.'

She plans to

- hand out a short questionnaire at a Women's Institute meeting,

Martha wants to compare men buying newspapers against women buying newspapers but is only giving the questionnaire to women.

Example: A survey is to be carried out to find the popularity of buying books with various age groups of the general population.
The survey is carried out by asking people question as they come out of a book shop.
Two questions from the survey questionnaire are shown below.

1. How old are you?
   Put a tick in the box.
   under 20 ☐
   20 to 30 ☐
   30 to 40 ☐
   older than 40 ☐

2. Do you buy books?
   Put a tick in the box.
   Yes ☐
   No ☐

a) Explain why this may be a biased survey.
The survey is being carried out outside a book shop therefore the answer to question 2 is more likely to be yes.

b) State a criticism about the design of question 1 in the survey.
The groups *under 20* and *older than 40* are too large. The intervals should be more spaced out.

Mathematics

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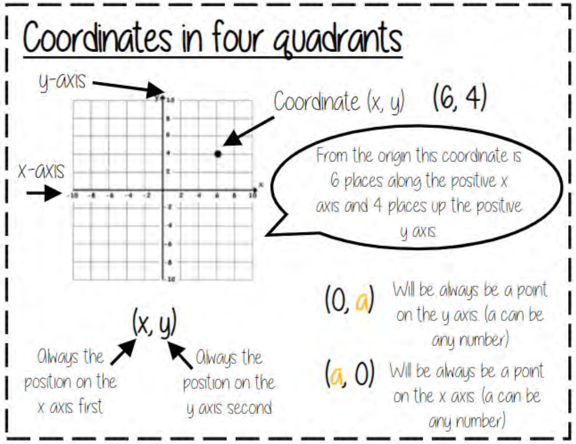
Unit 35

Straight Line Graphs

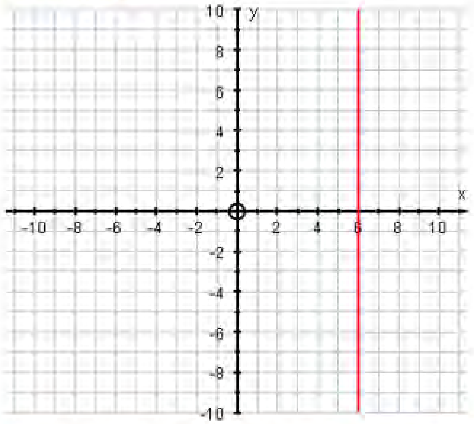
Recap: How to plot coordinates:



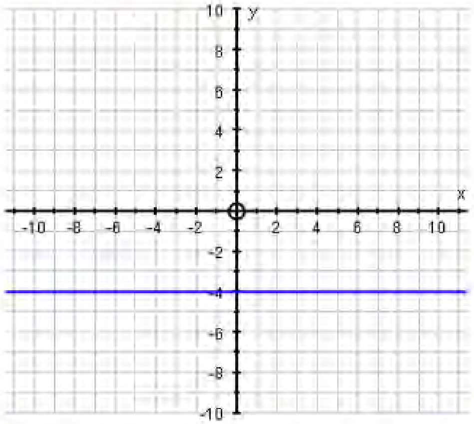
Graphs of $x = ?$ and $y = ?$

You need to learn how to recognise and draw horizontal and vertical lines.

Examples:



Every single point on this line has an x co-ordinate of 6, so the equation of the line is: **x = 6**



Every single point on this line has a y co-ordinate of -4, so the equation of the line is: **y = -4**

Note: The equation of the x axis is **y = 0**, and the equation of the y axis is **x = 0**.

What Does the Equation of a Straight Line Actually Mean?

The equation of a straight line is just a way of writing the relationship between the x coordinates and the y coordinates that lie on that line.

Example: $y = 2x - 1$

This says that the relationship between all the x coordinates and all the y coordinates is "take the x coordinate, multiply it by 2, subtract 1, this gives the y coordinate".

So, if you had these coordinates $(5, 9)$ then it is on the line (5 x 2 – 1 = 9 which is the y coordinate), but if you had the coordinates $(3, 2)$ then it is not on the line (3 x 2 – 1 = 5 which is not the y coordinate).

You end up with a straight line that goes through all the coordinates which share that relationship.

Mathematics

Foundation

Unit 35

Drawing Straight Line Graphs from their Equation

As well as graphs of horizontal and vertical lines, there are also graphs of diagonal lines.

Method for Drawing Straight-Line Graphs

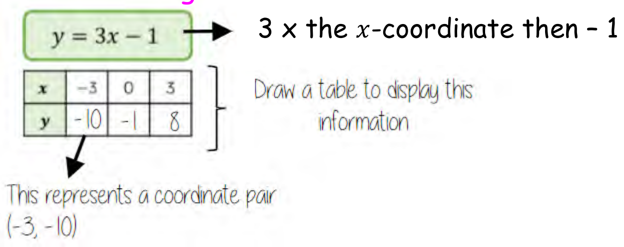
1. If the question does not give you values of x to use, then choose sensible values of x (A good choice of x values are 0, 1 and 2. This will show you the direction of the line. You need **at least** 3 values of x but choosing 4 (values -1, 0, 1 and 2) would make it even better).

2. Carefully substitute each x value it into the equation to get your y values, be careful if substituting negative numbers.

3. Join up the points with a straight line

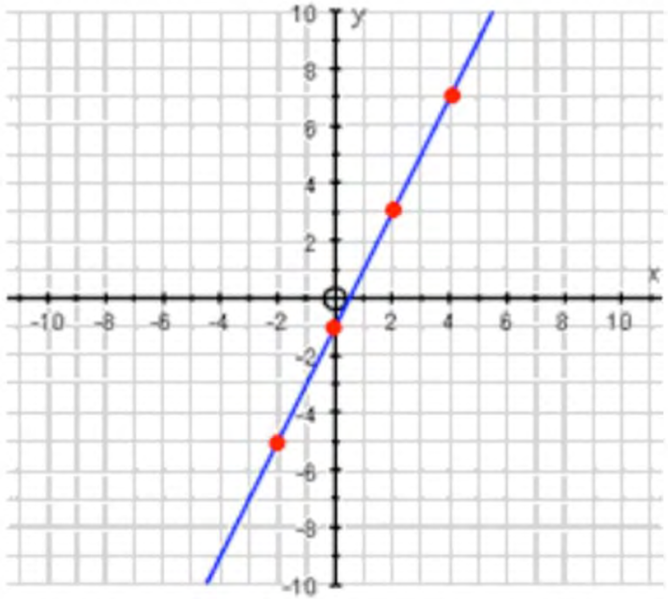
Note: The points should make a straight line, if one of your points does not lie on the straight line, check your substitution again.

Substituting:



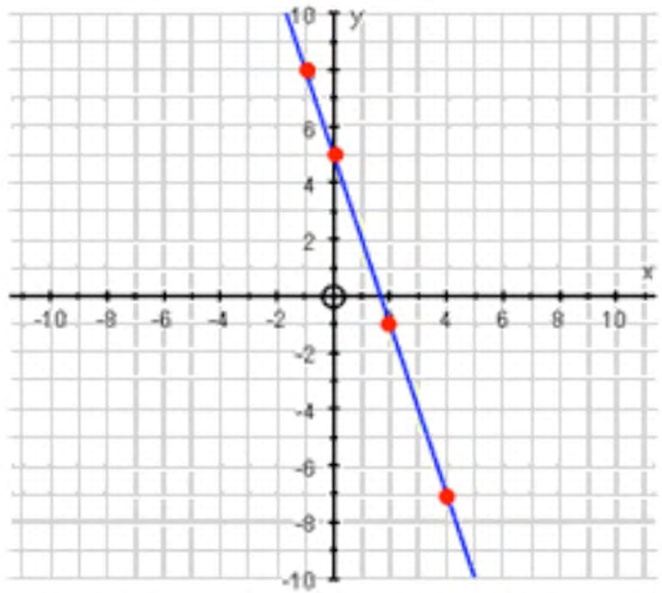
$y = 2x - 1$

x	0	2	4	-2
y	-1	3	7	-5



$y = -3x + 5$

x	0	2	4	-1
y	5	-1	-7	8



Mathematics

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Unit 36

Relative Frequency

Relative Frequency

Some probabilities can be estimated by doing experiments or trials, this is called relative frequency.

The more trials that are done (100+), the more accurate the estimated probability will be.

$$\text{Relative Frequency} = \frac{\text{number of times the event occurs}}{\text{total number of trials}}$$

Example 1:

A spinner is spun 100 times. The colour on the spinner is recorded after each spin. The table below shows the results recorded.

Colour	White	Green	Blue
Frequency	21	52	27

What is the relative frequency of spinning a green?

Number of times a green was spun

$\frac{52}{100} = 0.52$

Total number of spins

Example 2:

A dice is thrown 20 times. The number shown on the dice is recorded after each throw. The table below shows the results recorded.

Number shown on dice	1	2	3	4	5	6
Frequency	3	5	1	2	4	5

a) The relative frequency of throwing a 4 was calculates as $\frac{4}{20} = 0.2$.

What is the relative frequency of throwing a 2? Give your answer as a decimal.

$$\frac{5}{20} = \frac{1}{4} = 0.25$$

b) The number 1 was thrown 3 times in the first 20 throws. Using this fact, calculate how many times you would expect a 1 to be thrown when this dice is thrown 100 times?

$$\frac{3}{20} \times 100$$

$$\frac{3}{\cancel{20}_{1}} \times \cancel{100}^{5} = 3 \times 5 = 15 \text{ times}$$

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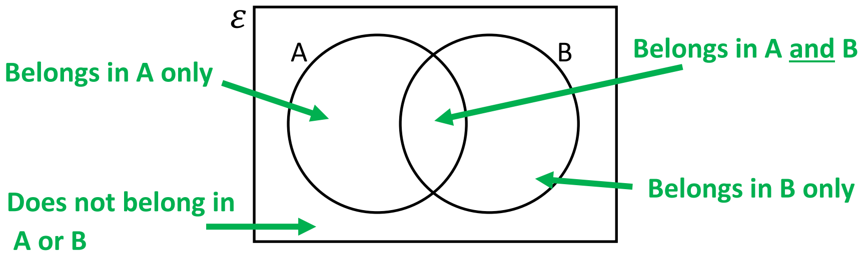
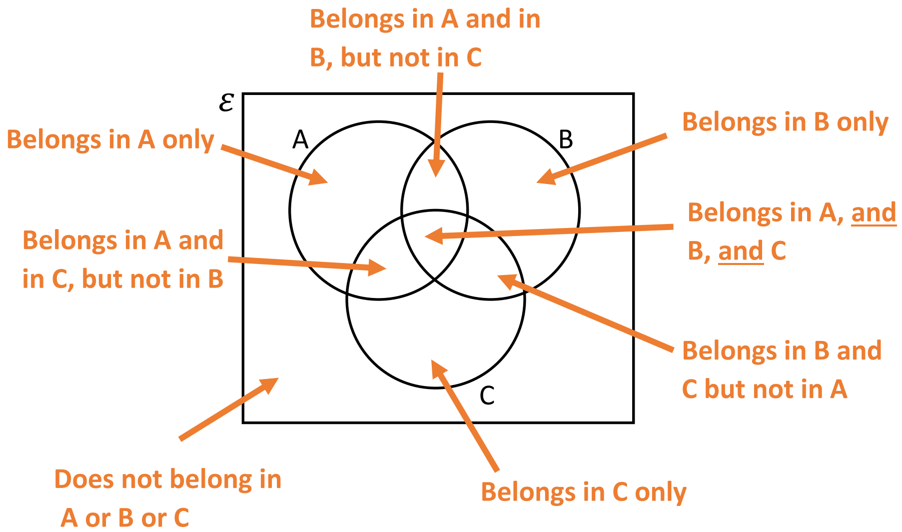
Venn Diagrams

A Venn diagram provides a means of classifying items of data which may or may not share common properties.
The universal set, ε, contains everything we are interested in at that time – it contains all the data we need to use for each individual question.

Drawing Venn Diagrams

A Venn diagram consists of 2 or 3 overlapping circles surrounded by a box.





Example: Display the following information in a Venn diagram.
Universal Set (ε): Integers between 5 and 15 inclusive
Set A: Multiples of 5
Set B: Odd numbers

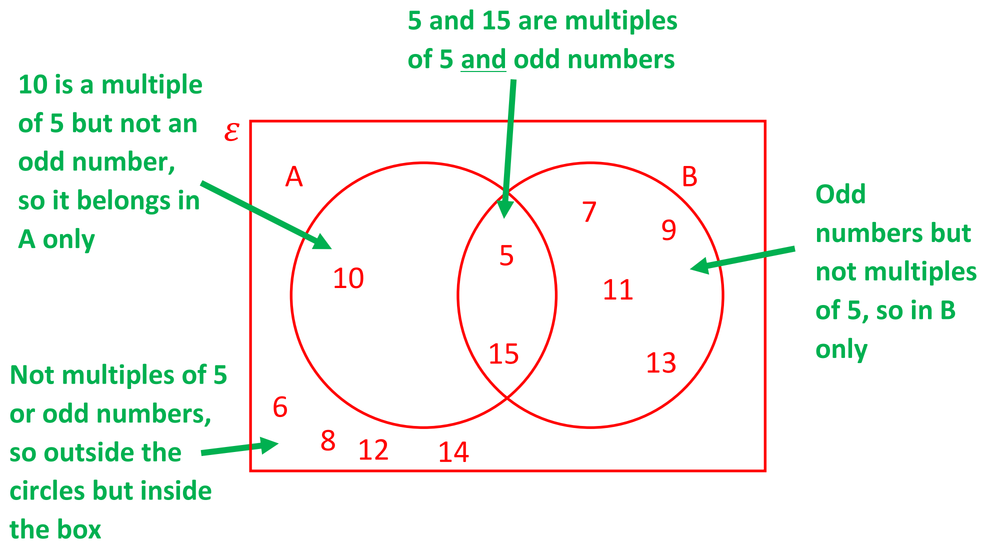
Whole numbers

All the numbers from 5 to 15 including the 5 and the 15

Step 1: Draw the Venn diagram, label the circles.

Step 2: From the numbers given:

- Write any that are multiples of 5 AND odd numbers in the centre area.
- Write any other multiples of 5 in the circle representing set A.
- Write any other odd numbers in the circle representing set B.
- Any numbers not yet used go outside the circles but inside the box.



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Venn Diagrams – Two Circles, Given the Number in the Overlapping Section

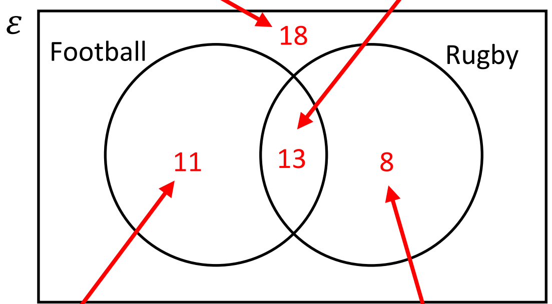
Example 1: In a group of 50 students:

- 24 play football
- 13 play both football and rugby
- 18 play neither football nor rugby

Complete the Venn diagram

Step 1: Fill in the number that goes outside the circles but inside the box.

Step 2: Fill in the centre section of the circles – the "both" section



Step 3: The whole of the football circle represents 24 students. There are already 13 of them in the overlapping area.
24 – 13 = 11.
There are 11 football students left.

Step 4: There are 50 students altogether.
18 + 13 + 11 = 42.
We have already used 42 students. 50 – 42 = 8.
There are 8 rugby students left.

Example 2: In a group of 140 students:
90 say Mathematics is their favourite subject

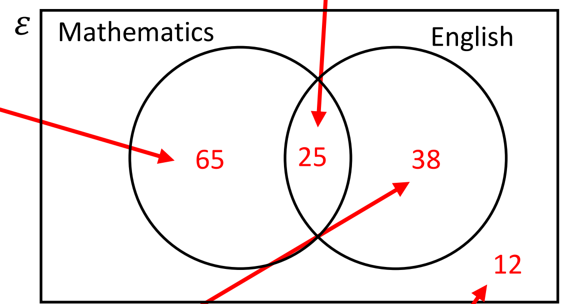
63 say English is their favourite subject

25 say both Mathematics and English are their favourite subjects

Complete the Venn diagram

Step 1: Fill in the centre section of the circles – the "both" section

Step 2: The whole of the Mathematics circle represents 90 students. There are already 25 of them in the overlapping area.
90 – 25 = 65. There are 65 Mathematics students left.



Step 3: The whole of the English circle represents 63 students. There are already 25 of them in the overlapping area.
63 – 25 = 38.
There are 38 English students left.

Step 4: There are 140 students altogether.
65 + 25 + 38 = 128.
We have already used 128 students.
140 – 128 = 12.
There are 12 students left.

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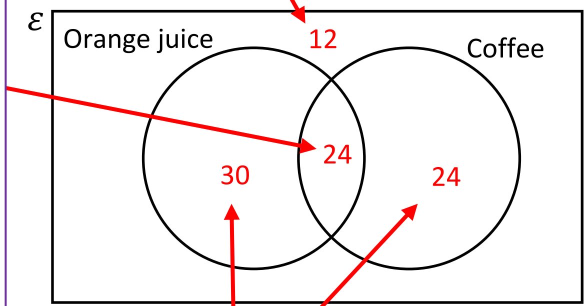
Venn Diagrams – Two Circles, Not Given the Number in the Overlapping Section

Example 3: There were 90 people at a breakfast buffet

- 54 drank orange juice
- 48 drank coffee
- 12 did not drink either orange juice or coffee

Complete the Venn diagram

Step 1: Fill in the number that goes outside the circles but inside the box.



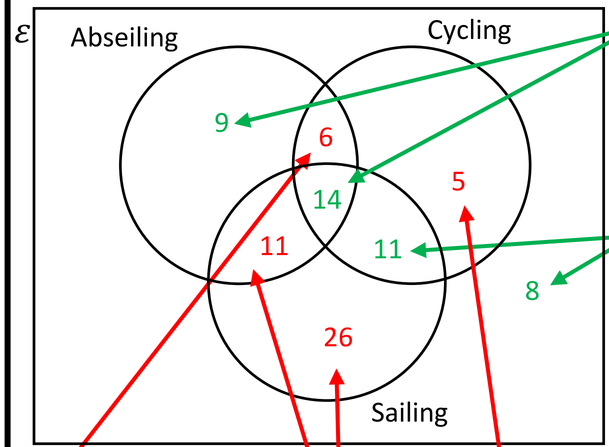
Step 2: We have already used 12 people.
90 – 12 = 78.
There are 78 people left to use.
If we add the number of people who drank orange juice and the number of people who drank coffee, we get
54 + 48 = 102.
This is more than the 78 we have left to use.
Find the difference
102 – 78 = 24
This difference goes inside the overlapping section – the "both" section

Step 3 and 4: The whole of the orange juice circle represents 54 people.
There are already 24 of them in the overlapping area.
54 – 24 = 30.
There are 30 orange juice people left.

The same with the Coffee people.
48 – 24 = 24.

Venn Diagrams – Three Circles

Example: 90 teenagers took part – or not – in various activities.
40 teenagers did abseiling
14 did all three activities and 8 did none of them
36 did cycling
9 did only abseiling
20 did abseiling and cycling
11 only did sailing and cycling



Step 1: We can fill these in straight away. 14 did all 3. 8 did none. 9 did **only** abseiling, meaning abseiling but not cycling and not sailing.
11 **only** did sailing and cycling, this does not include abseiling – it does not include the centre overlapping section

Step 3: Look at each circle in turn, always start with the circle which has just one section left to fill in

Step 2: We need to see if we can fill any other overlapping sections in first before we look at the circles as a whole.
20 did abseiling and cycling, this does include the centre overlapping section.
There are 14 teenagers already in the centre.
20 – 14 = 6

The cycling circle has one section left to fill in. 35 did cycling. This is the whole cycling circle.
6 + 14 + 11 = 31
We have already got 30 teenagers in the cycling circle.
36 – 31 = 5.

The abseiling circle has one section left to fill in. 40 did abseiling. This is the whole abseiling circle.
9 + 6 + 14 = 29
We have already got 29 teenagers in the abseiling circle.
40 – 29 = 11.

And now the sailing circle only has one section left to fill in.
We need to see how many teenagers are left.
9 + 6 + 5 + 11 + 14 + 11 + 8 = 64
90 – 64 = 26.

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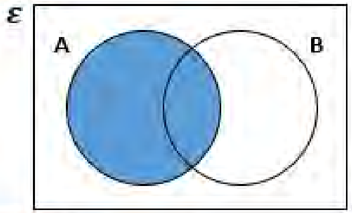
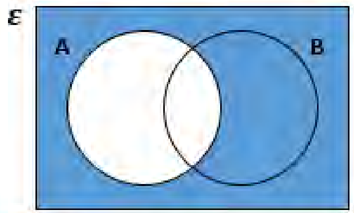
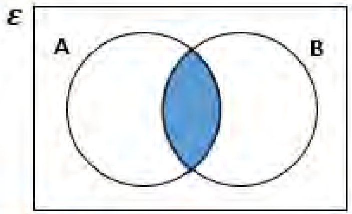
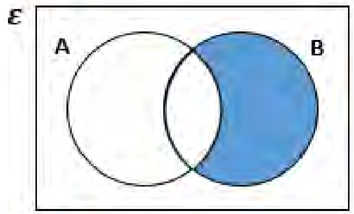
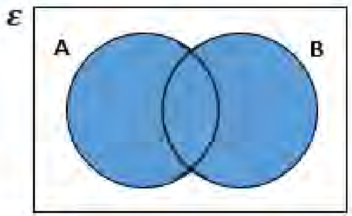
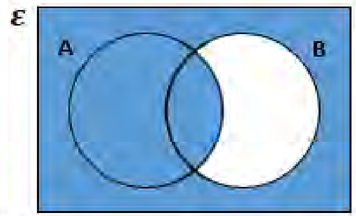
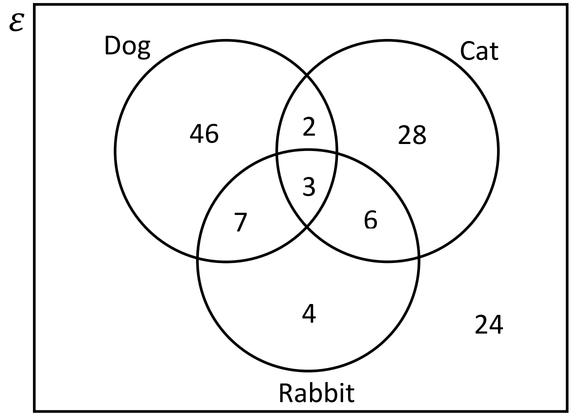
Set notation

A	Everything in set A	A'	Everything not in set A (The complement of A)
			
A ∩ B	A intersect B Everything in both	A' ∩ B	Everything in B and not A
			
A ∪ B	A union B Everything in A or B	A ∪ B'	Everything in A or not in B
			

Probability and Interpreting Venn Diagrams

Example: A group of people were asked what pet they had. The answers are shown in the Venn diagram below. The universal set, ε, contains all the people in the group.



a) How many of the people had exactly one pet?

Exactly one pet means only a dog (46) or only a cat (28) or only a rabbit (4).

P(D **or** C **or** R) = 46 + 28 + 4 = 78

b) How many people had a cat and a dog?

A cat and a dog is the overlapping section between the dog circle and the cat circle, as the question does not say **only** a cat and a dog it includes the middle overlapping section as well, where all three circles overlap. P(C and D) = 2 + 3 = 5

c) One of the people is chosen at random. What is the probability that the person has a dog? The question does not say **only** a dog, so we are looking at the whole dog circle. 46 + 2 + 3 + 7 = 58.

The total number of people asked is

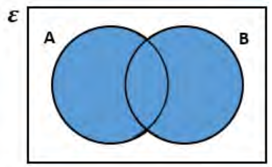
46 + 2 + 3 + 7 + 28 + 6 + 4 + 24 = 120

P(dog) = $\frac{58}{120}$ or $\frac{29}{60}$

d) One of the people is chosen at random. What is the probability that the person did not have a cat or a dog or a rabbit?

P(no pet) = $\frac{24}{120}$ or $\frac{1}{5}$

Example: Which of the following sets represents the **shaded** area in the Venn diagram below? Circle your answer



A $\quad B'$ $\quad A \cup B$ $\quad A \cap B$ $\quad A' \cap B$

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Unit 38

Compound Measures

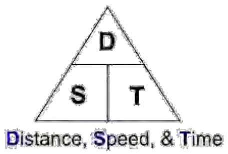
How to use Formula Triangles:

1) **COVER** the thing you want to find and **WRITE DOWN** what's left showing.
2) Now **SUBSTITUTE** in the things you know and **SOLVE**.

Speed

Speed is the distance travelled e.g. the number of km per hour or metres per second.

Formula Triangle:



$$Distance = Speed \times Time$$

$$Speed = \frac{Distance}{Time} \qquad Time = \frac{Distance}{Speed}$$

Example: A car travels 10 miles at 45 miles per hour. How long does this take?

Step 1: Use the formula triangle to write down the correct formula:

$$Time = \frac{Distance}{Speed}$$

Step 2: Substitute the values from the question:

$$Time = \frac{9\ miles}{36\ mph}$$

Step 3: Solve and check you have used the correct units:

$$Time = 0.25\ hours$$

If asked for in minutes: 0.25 x 60 = 15 minutes

Fuel Efficiency

Fuel efficiency is usually measured in miles per gallon, we use the following formulas:

$$distance\ travelled = fuel\ efficiency \times fuel\ used$$

$$fuel\ efficiency = \frac{distance\ travelled}{fuel\ used} \qquad fuel\ used = \frac{distance\ travelled}{fuel\ efficiency}$$

Example: Calculate the fuel efficiency of a car that has travelled 475 miles and used 9.5 gallons of fuel.

Step 1: Write down the correct formula from the ones above.

$$Fuel\ efficiency = \frac{distance\ travelled}{fuel\ used}$$

Step 2: Substitute the values from the question:

$$Fuel\ efficiency = \frac{475\ miles}{9.5\ gallons}$$

Step 3: Solve and check you have used the correct units:

$$Fuel\ efficiency = 50\ miles\ per\ gallon$$